# Long run risks, the factor structure of price dividend ratios and the cross section of returns*

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**Abstract**

I show that long run consumption risk models studied in the literature imply that log price dividend ratios (LPDs) of stocks have a strict factor structure, and that these factors predict consumption growth, dividend growth and consumption growth volatility. They further imply that the expected excess returns of assets are a linear function of the covariance of their returns with consumption growth, the factors and their innovations. Factor analysis of the LPDs of the 25 Fama-French size and book to market ratio sorted portfolios for the period 1943-2008 reveals two significant factors similar to the form postulated by Bansal and Yaron (2004). The first factor is related to consumption growth volatility and the second predicts dividend and real time consumption growth. The resulting LPD factor model is able to explain not only the cross section of excess returns of the 25 Fama-French portfolios, but also those of portfolios formed on the basis of size, book to market ratio, long term reversal, short term reversal and earnings to price ratio. The intercept terms in the cross sectional regressions are not significantly different from zero and the market prices of risk are significant and have the expected sign. However, the risk aversion of the representative agent implied by the estimated factor risk premia exceeds 40. The observed negative correlation between innovations to long term consumption growth and consumption growth volatility, when taken together with the long run risk model, implies that consumption growth should predict future excess market returns. This is confirmed in the data, both in and out of sample.

JEL classification codes : G12, G17

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# 1 Introduction

The observed cross-sectional variation of asset returns has presented a major challenge to asset pricing models (Mehra and Prescott 1985) (Fama and French 1992) (Fama and French 1993) (Fama and French 1995). In particular, the observed excess return of equities over the risk-free rate (i.e. the equity premium) (Mehra and Prescott 1985) and that of value stocks over that of growth stocks (Fama and French 1995) (i.e. the value premium) are much higher than expected from traditional asset pricing models. The historical real continuously compounded returns of various stock portfolios tabulated in table 1 illustrates this particularly clearly. From it, we see that the overall stock market has returned about 5% per annum more than the risk free asset while small value stocks have returned nearly 7% per annum more than small growth stocks and 10% more than the risk free asset. This is in spite of the fact that the betas of the assets are not very different from each other.

Among the standard asset pricing models, consumption based ones such as the consumption CAPM model (Breeden 1979) have performed particularly poorly. Despite this, such models have been popular in the asset pricing literature as economics suggests that consumption based risk must ultimately account for excess asset returns. In recent years, habit formation (Abel 1990) (Constantinides 1990) (Campbell and Cochrane 1999) and long run risk models (Bansal and Yaron 2004) (Bansal, Yaron, and Kiku 2007) (Bansal, Dittmar, and Kiku 2009) (Bansal, Dittmar, and Lundblad 2005) (Hansen, Heaton, and Li 2008) have revolutionized consumption based asset pricing and their study has been a fruitful area of research.

**Summary of historical returns of various stock portfolios**

| | Returns | | Div. growth rate | | $\beta_{Mkt}$ |
|---|---|---|---|---|---|
| | Mean (%) | $\sigma$ (%) | Mean (%) | $\sigma$ (%) | |
| Risk free rate | 1.20 | 2.3 | | | |
| Market | 6.11 | 18.0 | 1.51 | 5.9 | 1.00 |
| Growth (B1) | 4.65 | 21.0 | 0.49 | 15.5 | 1.09 |
| Value (B10) | 9.49 | 24.0 | 4.77 | 21.3 | 1.03 |
| Small cap (S1) | 7.42 | 27.5 | 4.16 | 15.1 | 1.08 |
| Large cap (S10) | 5.71 | 19.5 | 0.96 | 6.6 | 0.94 |
| Small growth (FF1) | 4.10 | 25.9 | -0.98 | 19.3 | 1.31 |
| Small value (FF3) | 11.17 | 22.4 | 6.51 | 15.6 | 1.02 |
| Large growth (FF4) | 5.61 | 18.5 | 1.05 | 10.1 | 1.03 |
| Large value (FF6) | 8.60 | 20.3 | 3.04 | 11.3 | 0.93 |

Table 1: Descriptive statistics of the real continuously compounded returns, logarithm of the price-dividend ratios and real dividend growth rates of ten interesting assets over the period 1950-2008.

In this paper, I focus my attention on long run risk models which do not impose co-integration between consumption and dividends, or, in other words, models of the type proposed by Bansal and Yaron (2004). These models have received wide attention in the recent literature and calibration based studies have shown that they have the potential to explain a wide variety of asset pricing puzzles such as the high equity premium, low level and volatility of the risk-free rate (Weil 1989) and high variance premium (Carr and Wu 2009) (Bollerslev, Tauchen, and Zhou 2008) (Dreschler and Yaron 2008) (Zhou and Zhu 2009).

The success of these models in explaining the value premium is unclear from the current literature. The calibration based study by Kiku (2006) and the estimation based studies by Bansal, Yaron, and Kiku (2007) (which ignores the exact form of the model in forming the stochastic discount factor) and Hansen, Heaton, and Li (2008) suggest that they perform very well but the more formal tests conducted by Constantinides and Ghosh (2008) show that the model proposed by Bansal and Yaron (2004) is rejected. The recent study of Ferson, Nallareddy, and Xie (2009) also shows that this model performs poorly out of sample.

In this paper, I study this issue by formulating a long run risk model which encompasses those proposed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) and which is capable of accommodating more complicated dynamics of the predictable consumption growth and volatility processes by incorporating additional factors. I show that it implies that log price-dividend ratios of assets have a factor structure with the factors being related to consumption growth and volatility and that the lagged values and innovations of these factors price assets according to these relations. Given that the returns and dividend growth of stock portfolios are only marginally related in the data,[1] their returns are approximately equal to the innovations of their log price dividend ratios. Hence, this relation also implies that stock returns have an approximate factor structure and that the long run risk model and the ICAPM (Merton 1973) are related. I proceed to show that standard factor analysis methods can be used to determine the number of factors in the data and to extract them.

The empirical analysis of this form of the model is complicated by the presence of regime shifts in the data. I find strong evidence of a structural break at around 1942 in the implied relationship between log price dividend ratios and future consumption and dividend growth (and, hence, the models proposed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009)). This implies that caution must be used in interpreting the results produced by tests of these models when the time period spans both regimes since these tests implicitly assume that the parameters are constant. Such tests are common in the literature (Bansal, Yaron, and Kiku 2007) (Constantinides and Ghosh 2008) (Ferson, Nallareddy, and Xie 2009) and they generally find more support for long run risk models when restricted to post war data, i.e. one regime. While it is well known that returns and dividends during the Great

[1] A recent study by Ball, Sadka, and Sadka (2009) does, however, provide some evidence that earnings and returns are linked in the cross-section.

Depression were qualitatively different from those after the war and previous studies by Nelson and Kim (1993) Pastor and Stambaugh (2001) Lettau and van Nieuwerburgh (2008) Welch and Goyal (2008) have identified structural breaks in returns and price dividend ratios, the identification of a structural break in the context of long run risk models is, as far as I am aware, original.[2] Due to the existence of this structural break, I restrict myself to post 1942 data in my analysis. This is equivalent to assuming that agents do not anticipate any further regime shifts in the future even though they have occured in the past.

After restricting myself to the post 1942 period, I use the twenty five Fama-French portfolios sorted on the basis of size and book to market ratio as the assets for the factor analysis of the log price dividend ratios. I use this set of assets as they are ubiquitous in the asset pricing literature, Parker and Julliard (2005) and Malloy, Moskowitz, and Vissing-Jørgensen (2009) find evidence that their returns are related to future consumption growth, data on them exist over a long time frame, a long time frame being essential for reliable estimation of the slow moving processes which are at the heart of long run risk models, and they do not have issues related to disappearance due to bankruptcy and mergers. While individual stocks provide a larger number of assets, the latter two considerations rule out their use. Individual stocks are also likely to be affected by factors which are unique to the firm or a small section of the economy but which are globally inconsequential and would thus introduce more noise into the estimation.[3]

By performing the factor analysis, I find that there are two significant factors in the log price dividend ratios of these assets by using an information criterion suggested by Bai and Ng (2008) and Connor and Korajczyk (2009). This finding provides an empirical foundation for the number of persistent factors, a number which is simply assumed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009).[4] It also strongly suggests that the incorporation of a larger number of persistent factors into the model is not a fruitful area for future research.

I find that the factor form of the long run risk model performs well, both with regard to pricing as well as with regard to the relation between the factors, consumption growth volatility and the joint component of consumption and dividend growth. First, I find that the innovations of the two factors price not only the twenty five portfolios used in estimating them, but also three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio. Second, I find that one of the factors is strongly negatively related to consumption growth volatility and that, as the model implies, it's shocks have a positive price of risk.[5] Third, I find that the other factor is

[2]Ferson, Nallareddy, and Xie (2009) hypothesize that structural breaks could be the reason why they find that long run risk models perform better in post war data but do not present any evidence to support their conjecture.

[3]Vuolteenaho (1999) finds, for example, that variation in book to market ratios at the individual stock level is largely idiosyncratic.

[4]The third factor in (Zhou and Zhu 2009) has very little persistence and determining it's presence using this approach is not feasible. Since this factor has negligible impact on excess asset returns due to it's low persistence, ignoring it's presence does not affect my results.

[5]Boguth and Kuehn (2008) also find that stocks exposed to consumption volatility risk

positively related to future dividend (both market and cross-sectional) and real time consumption growth (consumption growth estimated using data available immediately after the period) and that, as the model implies, it's shocks have a positive price of risk.[6]

My analysis differs from that by Beeler and Campbell (2009) in using several log price dividend ratios instead of just the log market price dividend ratio. This explains the difference in their results and mine. In particular, Beeler and Campbell (2009) find that the log market price dividend ratio, by itself, only predicts consumption growth volatility but not future dividend or consumption growth. This result is unsurprising as I find that the first factor which is related to consumption growth volatility explains as much as 86% of the variance of the log price dividend ratios of the twenty five Fama-French portfolios. When the second factor is included, I find that the future dividend and real time consumption growth are also predictable. This fact, together with the finding by Zhou and Zhu (2009) that a non-persistent volatility factor (which is not possible to identify with my methodology) can explain both the lack of predictability of market volatility by the log market price dividend ratio and the variance premium, implies that most of the predictability issues raised by Beeler and Campbell (2009) can be resolved and do not constitute vital evidence against long run risk models.

I, however, find, as Beeler and Campbell (2009) and many others do, that consumption growth estimated using current, and thus revised, data is largely unforecastable by log price dividend ratios. (I do find some indicative evidence that it is predicted by the second factor after a gap of four to five years.) I argue that this finding is less important for long run risk models than the finding that real time consumption growth is predictable in the expected manner. This is because agents need to form an estimate of consumption growth with data in their possession and this estimate is likely to be closer to the one made with real time data. The hypothesis that real time consumption growth is more relevant than consumption growth is further reinforced by my finding that, of the two, the former is more strongly negatively related to future excess market returns than the latter. This negative relation is not implied by long run risk models but is consistent with them provided there is a negative correlation between the innovations of long term consumption/dividend growth and consumption growth volatility, or in other words, a positive correlation between innovations of the first and second factors. I do indeed find such a correlation in the data and it is consistent not only with the above relationship but also with the negative skewness of consumption growth. I also find that the ability of real time consumption growth to predict future excess market returns is robust and holds out of sample unlike most other excess market return predictors considered in the literature (Avramov 2002) (Welch and Goyal 2008). This is a particularly surprising result since consumption growth is generally not among the list of

have higher excess returns but using a different model and methodology.

[6]These relations are also found to hold out of sample by estimating the rotation matrix relating the log price dividend ratios to the factors with data until 1975 and examining the relations in post-1975 data using factors constructed with this rotation matrix.

quantities considered as predictors of future excess market returns.

It is interesting to note that the external habit formation model in (Campbell and Cochrane 1999) implies a negative relationship between consumption growth and future excess market returns which is stronger when real time estimates of the former are used. Hence, this finding, when considered alone, can be construed as evidence for external habit formation. Similarly, the identified long run component in consumption growth can also be construed as evidence of internal habit formation (Grishchenko 2009). A separate finding of mine that can also be construed as evidence for both habit formation and long run risk models is that the first difference of consumption growth is able to explain the cross section of stock returns surprisingly well.[7] Hence, the findings reported in this paper, while providing evidence in favor of long run risk models, can also be interpreted as providing some evidence in favor of habit formation and do not rule it out. A full analysis of the relative merits of both types of models is, however, beyond the scope of this paper.

## 2 Related Literature

Other studies of long run risk models and the cross section of stock returns include those by Bansal, Yaron, and Kiku (2007), Constantinides and Ghosh (2008) and Ferson, Nallareddy, and Xie (2009). These studies assume that the number of factors is fixed and equal to two and that there is no measurement error or noise in the real risk-free rate and the market price dividend ratio. This study explicitly incorporates possible noise and measurement error in price dividend ratios and thus, given the null of the model, estimates the factors more accurately. This is done at the cost of estimating the stochastic discount factor from the form of the model rather than as a function of the time series parameters of the consumption and dividend processes and the log price-dividend ratios as by Constantinides and Ghosh (2008).[8] This is, however, not a significant cost since the stochastic discount factor is highly sensitive to the time series parameters which themselves cannot be estimated accurately as shown by Constantinides and Ghosh (2008) and, more importantly, because such a procedure is very sensitive to the precise form of the model. For example, as shown by Marakani (2009b), the results in Constantinides and Ghosh (2008) do not generalize to the trivial extension of the long run risk model of Bansal and Yaron (2004) by Bansal, Yaron, and Kiku (2007). Ferson, Nallareddy, and Xie (2009) also take a position similar to that in this paper and do not estimate the time series parameters of the consumption and dividend processes. As pointed out in the introduction, another difference between these studies and the current paper is that they do not check whether the parameters of the underlying

[7] Details of this result are available upon request from the author.

[8] Bansal, Yaron, and Kiku (2007) use a mixed approach in this regard, with the market prices of risk being taken as functions of the time series parameters as specified by the model. They, however, do not impose the restriction that this implies when estimating the priced shocks.

model are constant during the estimation period.

An alternative long run risk formulation which I do not explore in this paper is due to Hansen, Heaton, and Li (2008).[9] Hansen, Heaton, and Li (2008) show that the value premium, but not the equity premium, can be explained using this formulation and that a relatively high risk aversion value of around 30 is required for this purpose.[10] Malloy, Moskowitz, and Vissing-Jørgensen (2009) show that a closely related formulation is capable of explaining both the equity and value premia when stockholder consumption is used and that a relatively low relative risk aversion value of about 15 is sufficient for this purpose. As shown by Malloy, Moskowitz, and Vissing-Jørgensen (2009), the study of Parker and Julliard (2005) can also be cast into this framework. Despite the impressive results obtained using this approach, I do not explore it further because it does not easily accommodate stochastic volatility which, as shown by the studies of Bansal and Yaron (2004), Boguth and Kuehn (2008), Zhou and Zhu (2009) and Beeler and Campbell (2009), plays a crucial role in pricing assets when Epstein-Zin preferences are used.

Ludvigson, Chen, and Favilukis (2007) estimate the Epstein-Zin model directly without imposing any further assumptions by approximating the continuation utility with the use of splines and find that it is, with the use of stockholder consumption, able to explain the cross-section of stock returns with a modest risk aversion of 17.

# 3 Data

In this section, I describe the data used in this paper. Consumption data is obtained from the National Income and Product Accounts (NIPA) tables available at the BEA web site. Real annual per capita consumption is defined to be the nominal aggregate annual consumption of nondurables and services divided by the NIPA estimate of the mid-year population and deflated by the implicit personal consumption deflator.[11] Annual consumption growth is defined to be the first difference of the logarithm of this series. Quarterly consumption data is also obtained from NIPA and quarterly consumption growth is defined in a similar manner.

The proxy used for the nominal risk-free rate is the Fama 3 month T-bill rate taken from CRSP. As described in the analysis below, this is converted to

[9] While this formulation is derived using only a slight extension of the model of Bansal and Yaron (2004), it is essentially different in that consumption growth is not necessarily persistent. This is most clearly seen from the results of Malloy, Moskowitz, and Vissing-Jørgensen (2009) where top stockholder consumption growth, which is much less persistent than aggregate consumption growth, generates a much higher equity and value premium than the latter.

[10] If aggregate rather than per capita consumption is used, as in Hansen, Heaton, and Li (2008), the risk aversion required is only about 20. It is, however, standard to use per capita consumption in the data Marakani (2009a).

[11] Since I make use of data expressed in terms of chained dollars, I use a Tornqvist type index (Whelan 2000) to construct the implicit consumption deflator.

a real rate using the realized and past inflation as measured by the CPI. The CPI data is obtained from CRSP.

The market proxy used is the CRSP value-weighted index of all stocks listed on the NYSE, AMEX and NASDAQ. The construction of portfolios based on size and book-to-market ratios is as in (Fama and French 1993) and (Fama and French 1996). Data on the 6 ($2 \times 3$) and 25 ($5 \times 5$) portfolios sorted on the basis of both these characteristics as well as the two sets of ten portfolios (deciles) sorted on either characteristic is obtained from Ken French's web site. For out of sample testing, I use three sets of ten portfolios each formed on the basis of long term reversal, short term reversal and the earnings to price ratio. The long term reversal portfolios are formed monthly on the basis of stock's return over the past five years minus it's return over the past year. In other words, they are formed at time $t-1$ (time being indexed by month) by sorting stocks into ten portfolios according to their returns from $t-61$ to $t-13$. Similarly, short term reversal portfolios are formed at time $t-1$ by sorting stocks into ten portfolios based on their return from $t-2$ to $t-1$. The earnings to price based portfolios are formed at the end of June of year $t$ by sorting stocks into ten portfolios (using NYSE breakpoints) on the basis of their earnings to price ratios where earnings are earnings before extraordinary items during fiscal year $t-1$ and the price is the market capitalization at the end of December of year $t-1$. Returns on these thirty portfolios are also obtained from Ken French's web site.

The growth and value portfolios denote the bottom and top book to market ratio deciles respectively. Monthly dividends of the portfolios are calculated using the difference between the returns of the dividend reinvested and non-reinvested portfolios. The price-dividend ratio is calculated by dividing the real price of the non-reinvested portfolio by the sum of the lagged twelve real monthly dividends to account for the pronounced seasonality of the dividend series. For this purpose, the prices and dividends are deflated by the CPI. The use of nominal dividends and prices for this computation produces virtually identical results.

Real time consumption data is obtained from the web site of the Federal Reserve Bank of Philadelphia and is described in (Croushore and Stark 2001). The real consumption during a quarter is defined to be the sum of the real consumption of nondurables and services during that quarter. The real consumption during a year is defined to be the sum of the real consumptions during each quarter of that year. Real per capita consumption during a period is defined to be the real consumption during that period divided by the mid-period estimate of population. The real time annual per capita consumption growth for year $t$ is defined to be the difference between the logarithms of the real per capita consumptions during years $t$ and $t-1$ respectively as calculated using data of vintage Q1 of year $t+1$. To provide an example, the data set of Q1 1976 vintage is used to construct the real time annual per capita consumption growth for 1975. It is constructed by adding the real nondurables and services consumptions of Q1-Q4 1974 and Q1-Q4 1975, dividing each of them by the mid-year estimates of the population, and then taking the difference of the logarithms of the corresponding quantities. Similarly, the real time quarterly per capita consumption growth for quarter $t$ is defined to be the difference between the logarithms of the real per capita consumptions during quarters $t$ and $t-1$ respectively using data of vintage $t+2$. Data of vintage $t+2$ rather than $t+1$ is used for this purpose as the data is not always available by quarter $t+1$ (this is not a problem for annual consumption growth as Q4 consumption data is always available by mid Q1 of the following year). The real time quarterly per capita consumption growth for quarter $t$ is thus in the information set of the agents by quarter $t+2$ and can be used by them to predict the excess market return in quarter $t+4$ as postulated in section 8. The mid-period population estimates are not real time and are from current data since real time data for them is not available. The results in section 8 are, however, robust to using the fully real time aggregate consumption growth data.

# 4 The long run risk model

I consider the following long run risk model which is general enough to accommodate the models proposed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) as special cases. (As written below, the model's volatility process requires a slight modification to accommodate the first two models. This is not of crucial importance as using a sum of a square root and an Ornstein-Uhlenbeck process for each volatility factor will allow the accommodation of all three models while leaving the central result unchanged. I do not use such a process in order to keep the model relatively simple. I also note that it is easy to add further families of factors such as persistent dividend growth without changing the central result but do not do so for the same reason. Even jumps in $X$ and $V$ can be included without affecting the principal theoretical and empirical results. Hence, the empirical analysis, while being straightforward to implement, actually accommodates a much more general class of models than that indicated.) Let $c$, $X_i, 1 \leq i \leq n$ and $V_j, 1 \leq j \leq m$ be the log consumption process, $n$ processes that determine it's conditional growth rate and $m$ processes that determine it's conditional growth rate volatility respectively. Let $d_l, l \leq 1 \leq L$ be the log dividend processes of $L$ portfolios (in general, the lower case variables correspond to the logarithm of the upper case variables). I assume that these quantities follow the processes

$$\begin{aligned} c_{t+\Delta t} =& c_t + \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t + \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}}\,(W_{t+\Delta t} - W_t) \\ & - \sum_{k=1}^{m} \varphi_{w,k}\sqrt{V_{k,t}}\,(Z_{k,t+\Delta t} - Z_{k,t}) \end{aligned} \tag{1}$$

$$X_{i,t+\Delta t} = X_{i,t}(1-\alpha_i\Delta t) + \varphi_{i,x}\sqrt{\sum_{j=1}^{m}\delta_{x,i,j}^2 V_{j,t}}\,(Y_{i,t+\Delta t} - Y_{i,t}), 1 \leq i \leq n \tag{2}$$

$$V_{i,t+\Delta t} = V_{i,t} - \kappa_i(V_{i,t} - \bar{V}_i)\Delta t + \sigma_i\sqrt{V_{i,t}}\,(Z_{i,t+\Delta t} - Z_{i,t}), 1 \leq i \leq m \tag{3}$$

$$
\begin{aligned}
d_{l,t+\Delta t} = & d_{l,t} + \left(\mu_l + \sum_{i=1}^{n} \phi_{l,i} X_{i,t}\right)\Delta t \\
& + \pi_{l,d}\left(\Delta c_{t+\Delta t} - \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t\right) \\
& + \sum_{i=1}^{n} \pi_{i,l,x}(X_{i,t+\Delta t} - X_{i,t}(1-\alpha_i \Delta t)) \\
& + \sum_{j=1}^{m} \pi_{j,l,w}\sigma_j \sqrt{V_{j,t}}\,(Z_{j,t+\Delta t} - Z_{j,t}) \\
& + \sqrt{\sum_{k=1}^{m} \delta_{l,d,k}^2 V_{k,t}}\,(B_{t+\Delta t} - B_t)
\end{aligned} \tag{4}
$$

where $W$, $Y_i, 1 \le i \le n$, $Z_j, 1 \le j \le m$ and $B$ are independent Wiener processes and $\sum_{i=1}^{m} \delta_{c,i}^2 = \sum_{j=1}^{m} \delta_{x,i,j}^2 = \sum_{k=1}^{m} \delta_{l,d,k}^2 = 1$ (these variables are necessary to ensure that the market volatility is decoupled to the consumption growth volatility as is the case in the data (Zhou and Zhu 2009)). The basic time interval of the process is assumed to be the same as that for which consumption is observed. This ensures that the stochastic discount factor can be related to the innovations in the processes $c$, $X$ and $V$. If the basic time interval of the process is smaller than that for which consumption is observed, time aggregation prevents the calculation of the stochastic discount factor as shown by Bansal, Yaron, and Kiku (2007) (though, given a specific model, reasonable approximations can be made as in (Marakani 2009b)). I note that at least some of the $\alpha_i$ and $\kappa_i$ must be small for the risks to be long lived and carry a high price.

One important point of departure from convention in the above equations is that consumption here is defined as a rate so that consumption from time $t$ to $t + \Delta t$ is $C_{t+\Delta t}\Delta t$ and log consumption from $t$ to $t + \Delta t$ is $c_{t+\Delta t} + \log \Delta t$. This makes no difference in the above equations since the log consumption is just offset by a constant, but it does ensure that the continuous time limit exists and, as shown in the appendix, makes it easy to obtain it as well.

This long run risk process, when written in continuous time, incorporates the one proposed by Zhou and Zhu (2009) as a special case (specifically with $n = 1$, $m = 2$ and $\varphi_{w,i} = 0, 1 \le i \le m$). When the volatility process (3) is modified to an Ornstein-Uhlenbeck one plus a constant by modifying the second term to $\sigma_{i,w}\Delta Z_{i,t+1}$ and the final term in (1) to $-\sum_{k=1}^{m} \varphi_{w,k}(Z_{k,t+\Delta t} - Z_{k,t})$, it incorporates the ones proposed by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007) as special cases (specifically with $n = 1$, $m = 1$, $\varphi_{w,i} = 0, 1 \le i \le m$, $\pi_{i,l,x} = 0, 1 \le i \le n, 1 \le l \le L$ and $\pi_{i,l,w} = 0, 1 \le i \le m, 1 \le l \le L$).

Consumers in the model have Epstein-Zin preferences (Epstein and Zin 1989)

$$
U_t = ((1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \tag{5}
$$

with $\gamma > 1/\psi$. This implies that they prefer early resolution of uncertainty and that persistent consumption growth and volatility shocks have a high market price of risk. This high price of risk results in a high equity premium and low risk-free rate.

## 5 Factor structure of log price-dividend ratios

In the appendix, I show, using an approach similar to that by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007), that this long run risk model implies that

$$\log\left(\frac{P_{l,t}}{D_{l,t}}\right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i} X_{i,t} + \sum_{j=1}^{m} A_{2,l,j} V_{j,t} \tag{6}$$

where $P_{l,t}$ is the price of portfolio $l$ and $A_{1,l,i} = \frac{(\phi_{l,i}-1/\psi)\Delta t}{1-\nu_{1,l}(1-\alpha_i \Delta t)}, 1 \leq i \leq n$ ($\nu_{1,l}$ being a log-linearization constant which is endogenously determined in the model). This generalizes the equivalent results by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) to the situation when there are multiple state variables describing predictable consumption growth and consumption growth volatility. The appendix also shows how to obtain expressions for $A_{2,l,j}, 1 \leq j \leq m$.[12]

Since the real risk-free rate can be viewed as a special type of dividend-price ratio, it also follows that

$$r_{f,t} = A_{0,f} + \sum_{i=1}^{n} A_{1,f,i} X_{i,t} + \sum_{j=1}^{m} A_{2,f,j} V_{j,t} \tag{7}$$

where $A_{1,f,i} = 1/\psi$.

In the appendix, I show that the log stochastic discount factor for this model is given by

$$\begin{aligned} m_{t+\Delta t} =& \Delta t \left(\Gamma_0 + \sum_{i=1}^{n} \Gamma_{1,i} X_{i,t} + \sum_{j=1}^{m} \Gamma_{2,j} V_{j,t}\right) \\ & - \lambda_c \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}} (W_{t+\Delta t} - W_t) \\ & - \sum_{i=1}^{n} \lambda_{x,i} \sqrt{\sum_{j=1}^{m} \delta_{x,i,j}^2 V_{j,t}} (Y_{i,t+\Delta t} - Y_{i,t}) \\ & - \sum_{j=1}^{m} \lambda_{v,j} \sqrt{V_{j,t}} (Z_{j,t+\Delta t} - Z_{j,t}) \end{aligned} \tag{8}$$

where $\Gamma_{1,i} = -1/\psi$, $\lambda_c = \gamma$ and $\lambda_{x,i} = \frac{\gamma - 1/\psi}{1-\nu_1(1-\alpha_i \Delta t)}$. The expression for $\lambda_{v,j}$ is complicated and does not directly concern us here, but it can be shown that,

[12] but does not explicitly do so for brevity.

as expected, $\lambda_{v,j} < 0$ if $\gamma - 1/\psi > 0$. The simple form of $\lambda_{x,i}$ implies that it can used together with a reasonable approximation for $1 - \nu_1 = \frac{\exp(\overline{c-w})\Delta t}{1+\exp(\overline{c-w})\Delta t} \approx \exp(\overline{c-w})\Delta t$ to determine $\gamma - 1/\psi$ once a component $X_i$ is identified. The estimation of $\nu_1$, which can at best be done heuristically, is a cost that has to be paid when the parameters are not explicitly specified.

In the case of no measurement error, (6) can be inverted to express the state variables $(X_i, V_j)$ as a linear combinations of log price dividend ratios. This enables the expression of the log stochastic discount factor as

$$m_{t+\Delta t} = \Delta t \left( \tilde{\Xi}_0 + \sum_{i=1}^{m+n} \tilde{\Xi}_{1,i} F_{i,t} \right) - \lambda_c (c_{t+\Delta t} - c_t) - \sum_{i=1}^{m+n} \lambda_{q,i} IF_{i,t+\Delta t} \qquad (9)$$

where $F_i$ and $IF_i$, $1 \leq i \leq n+m$ are the $n+m$ principal components of the log price dividend ratios (or, equivalently, any linear combination of $n+m$ log price dividend ratios) and their innovations respectively.

(6) clearly implies that the log price dividend ratios of assets follow a strict factor structure (up to the loglinear approximation) in the model.[13] Since log price dividend ratios are not exactly linear combinations of a small number of factors in the data, I use a slightly modified relation in my empirical work. This relation is

$$\log \left( \frac{P_{l,t}}{D_{l,t}} \right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i} X_{i,t} + \sum_{j=1}^{m} A_{2,l,j} V_{j,t} + \epsilon_{l,t} \qquad (10)$$

where $\epsilon_{l,t} \sim N(0, V_e)$ are i.i.d. $\epsilon_{l,t}$ can be thought of as measurement error or to consist of incompletely diversified idiosyncratic factors. In section 7, I show that, given the assumed error structure, singular value decomposition can be used to estimate the linear subspace that the $n+m$ factors span once $n+m$ is specified and that statistical tests suggested in the literature can be used to estimate $n+m$ from the data.

I emphasize that since there are measurement[14] or specification errors in all models (and their tests), which are only approximations to reality, the expression (9), which is exact in their absence, can be used as an approximation even when they exist and that studies which implicitly ignore them are not more accurate. In fact, it can be argued that controlling for measurement and specification errors is a better way of approaching this problem than implicitly ignoring them. It is in this spirit that I use the principal components of a number of log price dividend ratios rather than arbitrarily select a few of them. I thus minimize the effect of measurement errors and retain the desired information regarding the state variables of the model.

[13]The reader must note that these factors are different but related from the pricing factors discussed below. This terminology for both types of quantities is standard but unfortunate.

[14]Measurement errors are non-trivial when testing or estimating long run risk models in conventional ways since they involve the use of real risk free rates which are not directly observed for most of the data period and must be approximated using measures of expected or realized inflation.

I now derive two different factor beta pricing relationships using (9) and the standard relation

$$E_t[\exp(m_{t+\Delta t} + r_{c,t+\Delta t})] = 1 \tag{11}$$

each of them being based on a different approximation, and discuss their relative merits for asset pricing tests. The first relationship is derived by approximating the unconditional distribution of the log stochastic discount factor by a normal one and the second by assuming that conditional expected returns do not vary very much.

To derive the first factor pricing relationship, I approximate the unconditional distribution of the log stochastic discount factor (8) by a normal one. Using this approximation, I find that the unconditional version of (11) implies that

$$\begin{aligned} &E[r_{i,t+\Delta t}] - E[r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t}] \\ &- \frac{1}{2}\text{Var}[E_t[m_{t+\Delta t}]] + \text{Cov}(m_{t+\Delta t}, r_{i,t+\Delta t}) = 0 \end{aligned} \tag{12}$$

This, together with (9), provides the first factor beta pricing relationship with the factors being contemporaneous consumption growth together with the identified *lagged* principal components and their innovations. Explicitly, it can be written as

$$\begin{aligned} E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx &\beta_{\Delta c}\gamma_{\Delta c} + \sum_{i=1}^{n+m} \beta_{F_i}\gamma_{F_i} \\ &+ \sum_{j=1}^{n+m} \beta_{IF_j}\gamma_{IF_j} \end{aligned} \tag{13}$$

where $F_i$ and $IF_j$ stand for the lagged principal components or factors of the log price dividend ratios and their innovations respectively. The third term in the beta pricing relationship is required as the conditional mean of the log stochastic discount factor is time varying. However, it is constant for all assets and only changes the estimated zero beta rate. Further, it is seen from the relation

$$\begin{aligned} \text{Var}[E_t[m]] &= \sum_{i=1}^{n} \frac{1}{\psi^2}\text{Var}[X_i] + \sum_{j=1}^{m} \Gamma_{2,j}^2 \text{Var}[V_j] \\ &\leq \frac{1}{\psi^2}\text{Var}[\Delta c_{t+\Delta t}] + \sum_{j=1}^{m} \Gamma_{2,j}^2 \text{Var}[V_j] \end{aligned} \tag{14}$$

that it is likely to be small as the variance of consumption growth and it volatility are small and $\psi > 1$ in long run risk models.[15]

[15]For reasonable parameter values, numerical experiments verify this conjecture.

To derive the second factor pricing relationship, I use (11) to first derive the exact conditional relationship (exactness following from the conditional normality of continuously compounded returns and the log stochastic discount factor)

$$E_t[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}_t[r_{i,t+\Delta t} - r_{f,t}] + \text{Cov}_t(m_{t+\Delta t}, r_{i,t+\Delta t} - r_{f,t}) = 0 \quad (15)$$

This relationship in turn implies the unconditional relationship

$$\begin{aligned} &E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] - \frac{1}{2}\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]] + \\ &\text{Cov}[m_{t+\Delta t} - E_t[m_{t+\Delta t}], r_{i,t+\Delta t} - r_{f,t}] = 0 \end{aligned} \quad (16)$$

which, together with (9), provides another factor beta pricing relationship with the factors being contemporaneous consumption growth and the innovations of the identified principal components of the log price dividend ratios.[16] Explicitly, it can be expressed as

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \beta_{\Delta c}\gamma_{\Delta c} + \sum_{i=1}^{n+m} \beta_{IF_i}\gamma_{IF_i} \quad (17)$$

where $IF_i$ stand for the innovations to the principal components or factors of the log price dividend ratios. Note that the relationship requires an approximation for the third term in (16), which, unlike the third term in (12), varies cross-sectionally and biases the estimates of the market prices of risk. This term is also not necessarily small in long run risk models (using the parameter estimates and model in (Zhou and Zhu 2009), it is comparable in magnitude to the pricing errors obtained in my estimations) but is conventionally set to zero in the literature (see, for e.g., (Ferson, Nallareddy, and Xie 2009) and (Malloy, Moskowitz, and Vissing-Jørgensen 2009)). In my analysis, I also follow this convention and set it to zero but additionally proceed to compare the ensuing results with those generated by the use of (13) in order to examine if the use of this approximation is justifiable and find that there is reason to believe that it is not.

The major difference between the two factor beta pricing relationships (13) and (17) is that the former, in addition to the innovations of the principal components, also includes the lagged principal components themselves as factors. This difference is just a reflection of the fact that the third term in (16) is ignored when constructing the second beta pricing relationship, or in other words, of the assumption that the expected excess returns are not varying over time. In most models this is not a significant issue, but as I will show in the empirical section, there is reason to believe that it is important when long run risks are involved.

[16] I note that the expression $m_{t+\Delta t} - E_t[m_{t+\Delta t}]$ in the covariance term presents no practical issues as the removal of the principal components of the log price dividend ratios among the factors ensures that only the innovations in the stochastic discount factor remain in the pricing relationship.

I ignore the contemporaneous consumption growth factor in the empirical analysis below since it is well known that it does not explain the cross section of equity returns (such an approach is also used, for e.g., by Malloy, Moskowitz, and Vissing-Jørgensen (2009)). In unreported results, I find that it's inclusion does not affect any of the subsequent findings.

### 5.1 Relation to conventional factor models

I note that innovations of the log price dividend ratios are approximately the same as excess returns since the bulk of returns are not explainable by dividend growth (Cochrane 2005). Hence, the methodology involved here closely parallels that of standard factor analysis (Connor and Korajczyk 2009) but differs in the way the factors themselves are constructed. While standard factor analysis constructs factors from the returns themselves, this methodology pays more attention to returns that are not explained by contemporaneous dividend growth, or in other words, to the more interesting non-trivial part of returns. When the lagged principal components of the log price dividend ratios are included as factors, it extends factor analysis to include important but slow moving predictable components of excess returns which are usually ignored but which are also at the heart of long run risk models. Thus, we see that long run risk models can be related to factor models in the literature with the factors being not only the innovations of the long run risk components (which are analogous to returns), but also the components themselves (which are analogous to the price dividend ratios).

## 6 Structural break implied by the factors

When $n > 0$ (i.e. there is at least one factor which is a measure of future consumption growth), as is assumed in all such models in the literature, we can invert (6) to write

$$\sum_{i=1}^{n} X_{i,t} = \sum_{i=1}^{n+m} \alpha_i F_i \tag{18}$$

where the $\alpha_i$ are functions of the parameters of the model (Constantinides and Ghosh (2008) and Marakani (2009b), for e.g., derive $\alpha_i$ explicitly for their respective models), and which, given the constancy of the parameters, are constant over time. Since $\sum_{i=1}^{n} X_{i,t}$ is the predictable part of consumption growth and $\sum_{i=1}^{n} \phi_{l,i} X_{i,t}$ is the predictable part of dividend growth (which is thus also spanned by $F_i$), this implies that regressing consumption growth $c_{t+\Delta t} - c_t$ or dividend growth $d_{t+\Delta t} - d_t$on $n+m$ log price dividend ratios or the $n+m$ principal components must result in coefficients which do not change with time. I now show that this is not the case in the data for both $n+m$ equal to 2 and 3, the cases generally considered in the literature.[17]

[17]The result continues to hold for $n+m$ equal to 4 or 5. The evidence for this is available on request.

**Results of predictive regressions of consumption growth for 1930-2008 and 1950-2008 with two factors**

| Regressors | 1930-2008 | | | 1950-2008 | | |
|---|---|---|---|---|---|---|
| | Coefficient | S. E. | $R^2$ | Coefficient | S. E. | $R^2$ |
| Constant | −0·029 | 0·016 | | 0·013 | 0·012 | |
| Market $p_t - d_t$ | 0·0147 | 0·0047 | 29.86% | 0·0019 | 0·0036 | 0.54% |
| $r_{f,t}$ (realized inf.) | −0·255 | 0·05 | | −0·010 | 0·063 | |
| Constant | −0·031 | 0·018 | | 0·017 | 0·012 | |
| Market $p_t - d_t$ | 0·0150 | 0·0053 | 12.52% | 0·0003 | 0·0035 | 9.34% |
| $r_{f,t}$ (lagged inf.) | −0·135 | 0·063 | | 0·172 | 0·074 | |
| Constant | −0·024 | 0·018 | | 0·0090 | 0·0127 | |
| Value $p_t - d_t$ | 0·0025 | 0·0021 | 7.00% | 0·0030 | 0·0043 | 1.59% |
| Growth $p_t - d_t$ | 0·0087 | 0·0039 | | 0·0003 | 0·0035 | |
| Constant | 0·018 | 0·0024 | | 0·022 | 0·0021 | |
| Princ. comp. 1 (FF6) | 0·00445 | 0·0016 | 10.40% | 0·00050 | 0·0011 | 2.62% |
| Princ. comp. 2 (FF6) | 0·0032 | 0·0030 | | 0·0054 | 0·0047 | |

Table 2: Regressions of annual consumption growth on various sets of two log price dividend ratios (and risk free rates, which are just a kind of dividend price ratio) over the periods 1930-2008 and 1950-2008. The principal components are of the log price dividend ratios of the six Fama-French portfolios sorted on the basis of size and book to market ratio. The results for the real risk-free rate using realized inflation are more applicable as using expected inflation leads to very similar results for the period 1950-2008.

**Results of structural break tests for consumption growth with two factors**

| Regressors | Bai-Perron test | BIC values | p value of Chow test |
|---|---|---|---|
| Market $p_t - d_t$<br>$r_{f,t}$ (realized inf.) | 1 break at<br>(1944, 1948) | −389.10 (no break)<br>−416.99 (1 break) | $3.51 \times 10^{-6}$ |
| Market $p_t - d_t$<br>$r_{f,t}$ (lagged inf.) | 1 break at<br>(1945, 1948) | −371.65 (no break)<br>−424.05 (1 break) | $1.52 \times 10^{-5}$ |
| Value $p_t - d_t$<br>Growth $p_t - d_t$ | 1 break at<br>(1939, 1944) | −366.82 (no break)<br>−388.97 (1 break) | 0.0091 |
| Princ. comp. 1 (FF6)<br>Princ. comp. 2 (FF6) | 1 break at<br>(1939, 1942) | −369.76 (no break)<br>−409.23 (1 break) | 0.00015 |

Table 3: Results of structural break tests on the regressions in table 2. I perform the multiple regression structural break test from Bai and Perron (1998) and the more common Chow test often used to test for structural break in equity premium regressions (for e.g., see Welch and Goyal (2008)). The reported confidence intervals for the break data are 90% confidence intervals.

**Results of structural break tests for market dividend growth with two factors**

| Regressors | Bai-Perron test | BIC values | $p$ of Chow test |
|---|---|---|---|
| Market $p_t - d_t$<br>$r_{f,t}$ (realized inf.) | 1 break at<br>(1941, 1944) | −122.28 (no break)<br>−174.82 (1 break) | $5.72 \times 10^{-4}$ |
| Market $p_t - d_t$<br>$r_{f,t}$ (lagged inf.) | 1 break at<br>(1939, 1941) | −117.86 (no break)<br>−169.40 (1 break) | $2.53 \times 10^{-4}$ |
| Value $p_t - d_t$<br>Growth $p_t - d_t$ | 1 break at<br>(1942, 1945) | −114.46 (no break)<br>−167.56 (1 break) | 0.0025 |
| Princ. comp. 1 (FF6)<br>Princ. comp. 2 (FF6) | 1 break at<br>(1941, 1943) | −119.34 (no break)<br>−189.69 (1 break) | $3.88 \times 10^{-4}$ |

Table 4: Results of structural break tests on the two factor regressions. I perform the multiple regression structural break test from Bai and Perron (1998) and the more common Chow test often used to test for structural break in equity premium regressions (for e.g., see Welch and Goyal (2008)). The reported confidence intervals for the break data are 90% confidence intervals.

In table 2, I summarize the results of regressing consumption growth against various combinations of two log price dividend ratios over the time periods 1930-2008 and 1950-2008. The first two regressions involve regressing against the log market price dividend ratio and continuously compounded real risk free rate where the conversion to a real rate is done using realized inflation in the first regression and lagged inflation in the second regression. The third regression uses the log value and growth price dividend ratios as regressors while the fourth uses the first two principal components of the log price dividend ratios of the six Fama-French portfolios sorted on the basis of size and book to market ratio.

The change in time window from 1930-2008 to 1950-2008 results in large and significant changes to the regression coefficients for all the combinations of log price dividend ratios considered. This is a very surprising result as almost three quarters of the data points in the former period are present in the latter and it implies that it is highly unlikely that the regression coefficient during 1930-1949 is the same as that during 1950-2008. Similar results hold when real market dividend growth is used as the regressand.

Formal testing of the hypothesis that the regression coefficients (for both consumption and dividend growth) are constant confirms this conjecture. The results of performing the test of Bai and Perron (1998) (using the algorithm by Bai and Perron (2003) and as implemented by Zeileis, Leisch, Hornik, and Kleiber (2002), Zeileis, Kleiber, Krämer, and Hornik (2003) and Zeileis (2006)) as well as the Chow test for structural breaks for the regression of consumption and dividend growth against two log price dividend ratios are summarized in tables 3 and 4 respectively. The tabulated results clearly show that we can reject the hypothesis that the coefficients have been constant over the entire time period at a very low level of significance and also that they changed somewhere in the period 1940-1945. Similar results hold when considering 2 year consumption and real market dividend growth or when twice lagged predictors are used to account for time aggregation thus confirming the robustness of the evidence. Due to the difficulties of accounting for inflation when constructing the real risk free rate, it seems more prudent to use the results involving only the price dividend ratios of equity portfolios. From them, we can conclude that 1942 is a reasonable upper bound for the structural break.

**Results of predictive regressions of consumption growth for 1930-2008 and 1950-2008 with three factors**

| Regressors | 1930-2008 | | | 1950-2008 | | |
|---|---|---|---|---|---|---|
| | Coefficient | S. E. | $R^2$ | Coefficient | S. E. | $R^2$ |
| Constant | −0·033 | 0·017 | | 0·012 | 0·013 | |
| Market $p_t - d_t$ | 0·0146 | 0·0047 | 30.21% | −0·0042 | 0·0074 | 2.18% |
| $r_{f,t}$ (realized inf.) | −0·255 | 0·052 | | 0·029 | 0·075 | |
| Value $p_t - d_t$ | 0·0011 | 0·0018 | | 0·0070 | 0·0073 | |
| Constant | −0·038 | 0·019 | | 0·014 | 0·012 | |
| Market $p_t - d_t$ | 0·0148 | 0·0053 | 13.69% | −0·0105 | 0·0064 | 15.43% |
| $r_{f,t}$ (lagged inf.) | −0·145 | 0·064 | | 0·229 | 0·077 | |
| Value $p_t - d_t$ | 0·0021 | 0·0020 | | 0·0122 | 0·0061 | |
| Constant | −0·030 | 0·019 | | 0·0099 | 0·0127 | |
| Market $p_t - d_t$ | 0·0115 | 0·0148 | 7.75% | −0·0121 | 0·0122 | 3.33% |
| Value $p_t - d_t$ | 0·0015 | 0·0025 | | 0·0084 | 0·0069 | |
| Growth $p_t - d_t$ | 0·0011 | 0·0105 | | 0·0062 | 0·0069 | |
| Constant | 0·019 | 0·0024 | | 0·023 | 0·0025 | |
| Princ. comp. 1 (FF6) | 0·00445 | 0·0016 | 10.83% | 0·00052 | 0·0011 | 3.49% |
| Princ. comp. 2 (FF6) | 0·0032 | 0·0030 | | 0·0082 | 0·0061 | |
| Princ. comp. 3 (FF6) | −0·0065 | 0·0109 | | 0·0060 | 0·0085 | |

Table 5: Regressions of annual consumption growth on various sets of three log price dividend ratios (and risk free rates, which are just a kind of dividend price ratio) over the periods 1930-2008 and 1950-2008. The principal components are of the log price dividend ratios of the six Fama-French portfolios sorted on the basis of size and book to market ratio. The results for the real risk-free rate using realized inflation are more applicable as using expected inflation leads to very similar results for the period 1950-2008 where expected inflation data is available.

In table 5, 6 and 7, I provide the analogous results when three log price dividend ratios are used. It is again seen that the coefficients are very different in the regressions for the periods 1930-2008 and 1950-2008. This again shows that the relationship between future consumption growth and log price dividend ratios is very different in the periods 1930-1949 and 1950-2008. The formal structural break tests also confirm this result. A second structural break during the early to mid 1980s is found in the consumption growth regression using the first three principal components of the log price dividend ratios of the six Fama-French portfolios but the significance of this break is quite low and this break is not confirmed when other log price dividend ratios are used. Further, since this break holds only holds for consumption growth and not dividend growth, it does not signify a break in the relation being investigated (since $\sum_{i=1}^{n} X_i$ is the common component of consumption and dividend growth).

**Results of structural break tests for consumption growth with three factors**

| Regressors | Bai-Perron test | BIC values | p value of Chow test |
|---|---|---|---|
| Market $p_t - d_t$ | 1 break at | $-388.40$ (no break) | |
| $r_{f,t}$ (realized inf.) | (1944, 1947) | $-413.53$ (1 break) | $9.67 \times 10^{-6}$ |
| Value $p_t - d_t$ | | $-411.8$ (2 breaks) | |
| Market $p_t - d_t$ | 1 break at | $-370.79$ (no break) | |
| $r_{f,t}$ (lagged inf.) | (1939, 1941) | $-424.16$ (1 break) | $2.38 \times 10^{-5}$ |
| Value $p_t - d_t$ | | $-416.52$ (2 breaks) | |
| Market $p_t - d_t$ | 1 break at | $-366.06$ (no break) | |
| Value $p_t - d_t$ | (1939, 1942) | $-398.45$ (1 break) | 0.00099 |
| Growth $p_t - d_t$ | | $-391.69$ (2 breaks) | |
| Princ. comp. 1 (FF6) | 2 breaks at | $-365.76$ (no break) | 0.0034 |
| Princ. comp. 2 (FF6) | (1939, 1941) | $-402.74$ (1 break) | |
| Princ. comp. 3 (FF6) | (1981, 1986) | $-404.47$ (2 breaks) | |

Table 6: Results of structural break tests on the regressions in table 5. I perform the multiple regression structural break test from (Bai and Perron 1998) and the more common Chow test often used to test for structural break in equity premium regressions (for e.g., see (Welch and Goyal 2008)). The reported confidence intervals for the break data are 90% confidence intervals.

**Results of structural break tests for market dividend growth with three factors**

| Regressors | Bai-Perron test | BIC values | $p$ of Chow test |
|---|---|---|---|
| Market $p_t - d_t$ | 1 break at | $-121.11$ (no break) | |
| $r_{f,t}$ (realized inf.) | (1941, 1943) | $-173.45$ (1 break) | $6.87 \times 10^{-4}$ |
| Value $p_t - d_t$ | | $-159.25$ (2 breaks) | |
| Market $p_t - d_t$ | 1 break at | $-118.06$ (no break) | |
| $r_{f,t}$ (lagged inf.) | (1941, 1943) | $-169.76$ (1 break) | $1.55 \times 10^{-4}$ |
| Value $p_t - d_t$ | | $-158.41$ (2 breaks) | |
| Market $p_t - d_t$ | 1 break at | $-115.29$ (no break) | |
| Value $p_t - d_t$ | (1942, 1944) | $-175.04$ (1 break) | $4.84 \times 10^{-4}$ |
| Growth $p_t - d_t$ | | $-156.83$ (2 breaks) | |
| Princ. comp. 1 (FF6) | 1 break at | $-117.01$ (no break) | $2.50 \times 10^{-4}$ |
| Princ. comp. 2 (FF6) | (1941, 1943) | $-184.61$ (1 break) | |
| Princ. comp. 3 (FF6) | | $-168.72$ (2 breaks) | |

Table 7: Results of structural break tests on the three factor regressions. I perform the multiple regression structural break test from Bai and Perron (2005) and the Chow test. The reported confidence intervals for the break data are 90% CI.

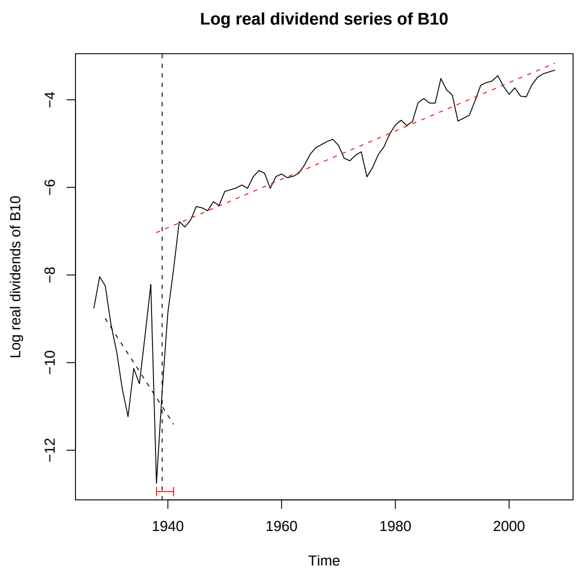


Figure 1: Structural break in the real dividend growth rate for value stocks.

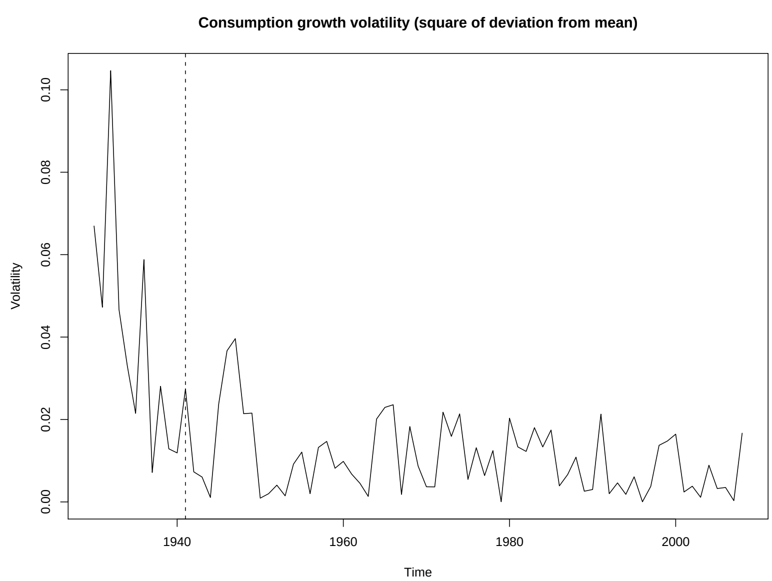


Figure 2: Structural break in consumption growth volatility.

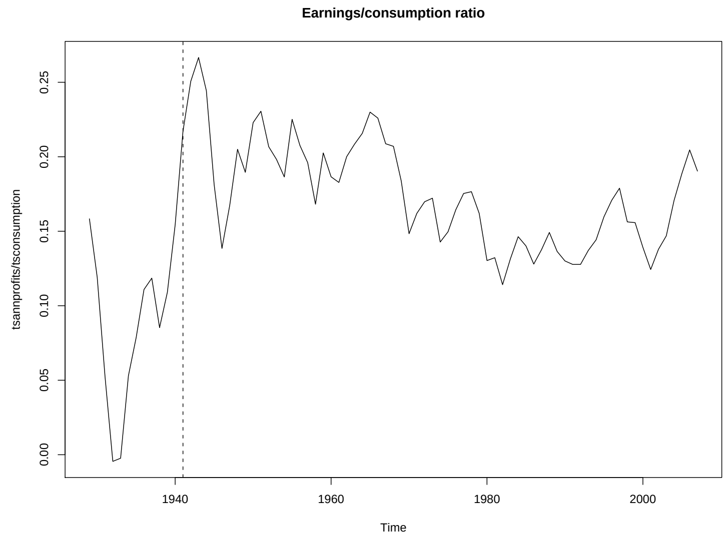


Figure 3: Structural break in the earnings to consumption ratio.

Other quantities of interest exhibit significant changes at around 1942 further giving credence to the hypothesis that a major regime shift occurred at about this time. A very clear structural break at about this time is seen in the real dividend growth rate for the value portfolio (figure 1). A similar break is also seen in consumption growth volatility defined as the square of the deviation of consumption growth from it's mean value (figure 2) which is much lower after it. The annual earnings to consumption ratio also exhibits a significant break in the early 1940s as seen from figure 3.[18]

Another interesting change that occurs around 1942 concerns the cross-sectional behavior of price dividend ratios. Before 1942, value stocks have much higher price dividend ratios than growth stocks as can be seen from figure 4. (Note that this is not due to the Great Depression as it is also true in 1927 and 1928 before it started.) As is well known, this is not the case after 1942. Since many models (such as those proposed by Lettau and Wachter (2007) and Santos and Veronesi (2009)) imply that value stocks endogenously have higher valuation ratios, the behavior of cross sectional price dividend ratios in the period before 1942 is puzzling. It is also puzzling in the context of long run risk models as they generally assume that value stocks have a higher leverage $\phi_{i,l}$ and are more sensitive to volatility shocks which result in them having particularly low price dividend ratios in bad times (low $X$, high $V$).

Since there is strong evidence that the parameters of the long run risk model could not have been the same before and after 1942, I only consider the post 1942 period in my analysis below and assume that consumers are myopic and do not

[18] These structural breaks are also confirmed by formal tests.

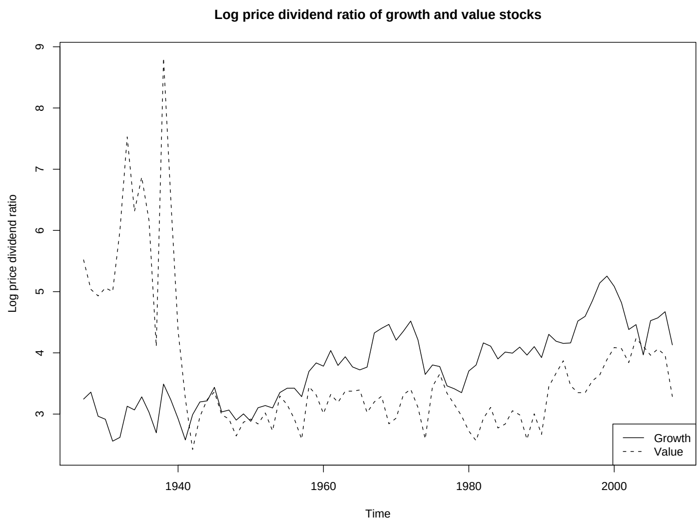


Figure 4: Time series of the price dividend ratios of value and growth stocks.

consider the possibility of regime change in the model. I defer the consideration of an extended model where the consumers are aware of possible regime shifts and leave it for future research.

# 7 Pricing implications and results

## 7.1 Construction of the principal components and their innovations

From (6), the problem of obtaining the factors of log price dividend ratios is, for a fixed number of factors $n+m$, equivalent to the problem of finding time series processes $F_i^{n+m}$ to solve

$$V(n+m) = \min_{\Lambda, F^{n+m}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left( X_{it} - \lambda_i^{n+m} F_i^{n+m} \right)^2 \qquad (19)$$

where $X$ is the matrix of demeaned log price dividend ratios, $N$ is the number of portfolios, $T$ is the length of the time series, $F$ are the factors and $\Lambda$ are the loadings of the individual log price dividend ratios on them (the superscript $n+m$ keeps track of the number of assumed factors). The equivalency of the two problems follows trivially from the assumption that the error term are i.i.d and Gaussian. (19) is the well studied standard factor analysis problem (Connor and Korajczyk (2009) is an excellent review of it's study).

Hence, the factors can be calculated by singular value decomposition of the matrix of de-meaned log price dividend ratios. This is equivalent to the more usual method of using the eigenvectors of the covariance matrix or directly solving (19), but is preferred because it is more numerically stable. The number of relevant factors $k = m + n$ is determined by using the information criterion

$$\underset{k}{\operatorname{argmin}}\, \mathrm{IC}_{p2} \equiv \underset{k}{\operatorname{argmin}} \left( \log V(k) + 2k \left( \frac{N+T}{NT} \right) \log \min(N, T) \right) \tag{20}$$

suggested by Bai and Ng (2008) (and also by Connor and Korajczyk (2009)). This method is known to be consistent when the number of quantities and the length of the time series become large. As pointed out by Bai and Ng (2008), traditional methods usually overestimate the number of factors that are present in the data.

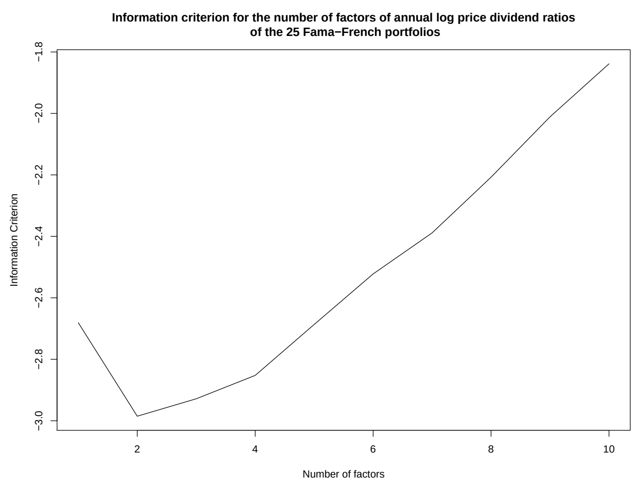


Figure 5: Information criterion as a function of the number of factors for the annual log price dividend ratios of the 25 Fama-French portfolios.

I carried out this procedure on the annual and quarterly log price dividend ratios of the 25 Fama French portfolios from 1943 (to account for the structural break) and 1947 (since quarterly consumption data is only available from this date) respectively. I found two significant factors in both series (as well in the monthly series of log price dividend ratios from 1947).[19] I plot the information

[19] The date change from 1943 to 1947 makes only a minimal difference to the estimated factors and the subsequent results remain largely unchanged even if the quarterly factors are estimated using data from 1943. Carrying out the procedure with the six Fama French portfolios over the period 1927-2008 leads to no optimal value for the number of factors (the information criterion monotonically decreases with $k$ for $k = 1 \ldots 6$). This shows the importance of accounting for the structural break before carrying out this analysis.

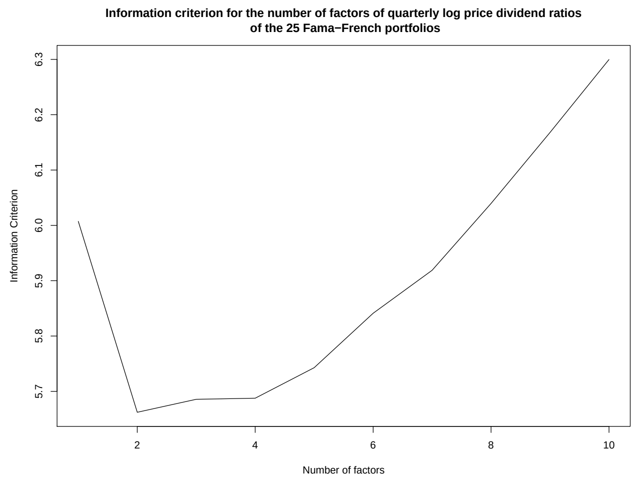


Figure 6: Information criterion as a function of the number of factors for the quarterly log price dividend ratios of the 25 Fama-French portfolios.

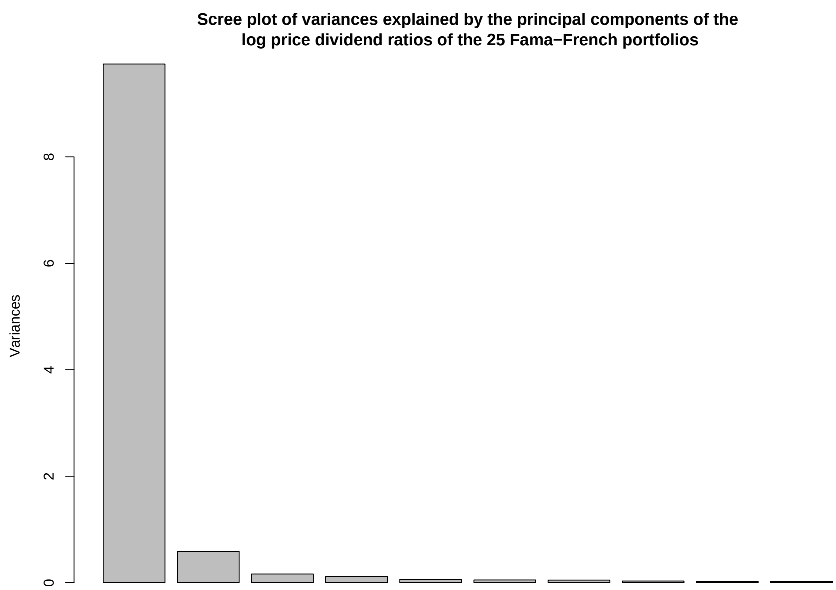


Figure 7: Variances explained by the first ten principal components of the annual log price-dividend ratios of the 25 Fama-French portfolios.

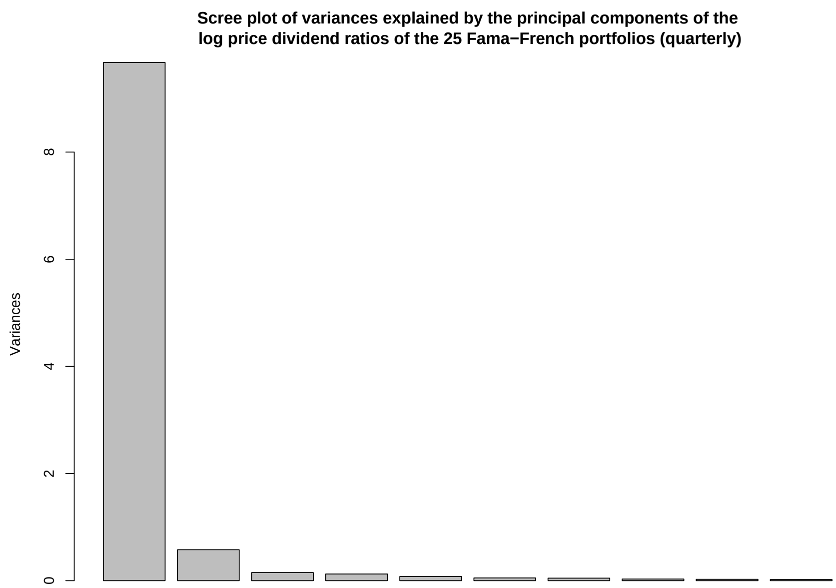


Figure 8: Variances explained by the first ten principal components of the quarterly log price-dividend ratios of the 25 Fama-French portfolios.

criterion I obtained as a function of the number of factors in figures 5 and 6, and the variances explained by the principal components in figures 7 and 8 respectively. As a check, I performed a similar analysis on the first differences of the log price dividend ratios as they should show a similar factor structure. I again found two significant factors at the quarterly frequency. The variances explained by the principal components of these first differences are plotted in figure 9 and the two factor structure is very evident in it.

I tabulate the rotations that relate the annual log price dividend ratios of the 25 Fama-French portfolios to their first two principal components, which I denote by $F_1^a$ and $F_2^a$, in tables 8 and 9 respectively. I tabulate the corresponding rotations for the principal components of the quarterly log price dividend ratios, which I denote by $F_1^q$ and $F_2^q$, in tables 10 and 11 respectively. I observe that the loadings of $F_1^a$ and $F_1^q$ on all the portfolios are positive. In contrast, I find that $F_2^a$ and $F_2^q$ load positively on large and value stocks and negatively on growth and small stocks. I thus expect them to be closely related to the cross sectional differences among the portfolios. I note that the estimated rotation matrices are essentially independent of the measurement frequency and that most of the small differences in the two sets of rotation matrices are due to the change in the starting date for the data used in their construction. Hence, where no fear of confusion arises, I ignore the measurement frequency and denote the two

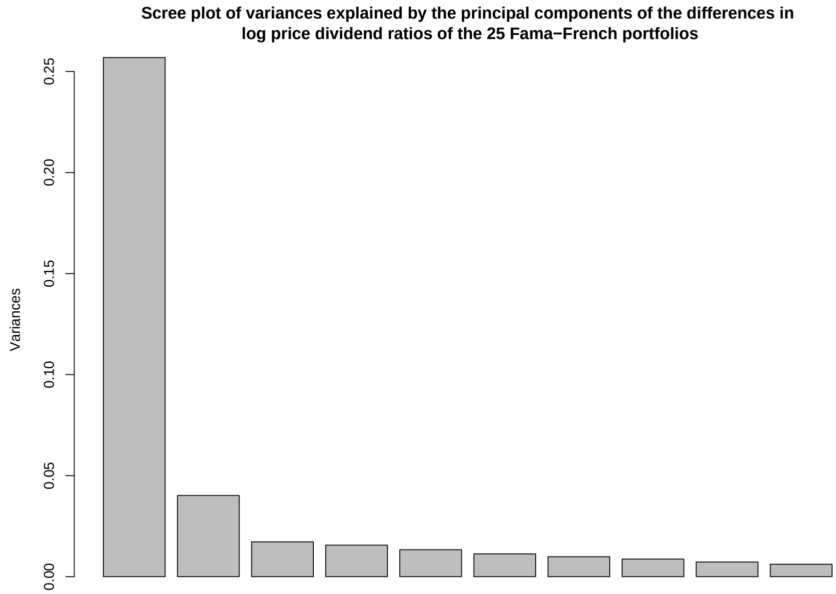


Figure 9: Variances explained by the first ten principal components of the first differences in the log price-dividend ratios of the 25 Fama-French portfolios.

principal components by $F_1$ and $F_2$ respectively.

The results obtained in the rest of the paper are largely unaffected by performing the principal component analysis by scaling the log price dividend ratios to make them each have unit variance or by scaling them each according to their residual variances.

**Rotation matrix for $F_1^a$**

| | | | | |
|---|---|---|---|---|
| 0·356 | 0·267 | 0·206 | 0·194 | 0·169 |
| 0·354 | 0·244 | 0·198 | 0·168 | 0·135 |
| 0·314 | 0·210 | 0·176 | 0·152 | 0·135 |
| 0·234 | 0·176 | 0·154 | 0·125 | 0·100 |
| 0·148 | 0·116 | 0·104 | 0·114 | 0·116 |

Table 8: The rotation matrix that relates the log annual price dividend ratios of the 25 Fama-French portfolios to their first principal component.

I estimated the innovations of the two identified principal components as the OLS residuals obtained on regressing them on $n$ lags of themselves, $n$ being the smallest value for which they were uncorrelated at the 10% level according to both the Ljung-Box and Durbin-Watson tests. $n$ was always found to be one

**Rotation matrix for $F_2^a$**

| | | | | |
|---|---|---|---|---|
| −0·458 | −0·140 | −0·082 | 0·028 | 0·104 |
| −0·289 | −0·055 | 0·089 | 0·194 | 0·211 |
| −0·177 | 0·025 | 0·185 | 0·231 | 0·221 |
| −0·064 | 0·091 | 0·170 | 0·302 | 0·243 |
| −0·005 | 0·077 | 0·186 | 0·275 | 0·315 |

Table 9: The rotation matrix that relates the log annual price dividend ratios of the 25 Fama-French portfolios to their second principal component.

**Rotation matrix for $F_1^q$**

| | | | | |
|---|---|---|---|---|
| 0·359 | 0·268 | 0·204 | 0·195 | 0·175 |
| 0·348 | 0·244 | 0·198 | 0·170 | 0·143 |
| 0·309 | 0·210 | 0·177 | 0·152 | 0·140 |
| 0·229 | 0·176 | 0·154 | 0·127 | 0·106 |
| 0·147 | 0·116 | 0·105 | 0·114 | 0·113 |

Table 10: The rotation matrix that relates the log quarterly price dividend ratios of the 25 Fama-French portfolios to their first principal component.

**Rotation matrix for $F_2^q$**

| | | | | |
|---|---|---|---|---|
| −0·506 | −0·155 | −0·080 | 0·019 | 0·066 |
| −0·265 | −0·046 | 0·092 | 0·191 | 0·168 |
| −0·153 | 0·029 | 0·192 | 0·241 | 0·198 |
| −0·041 | 0·099 | 0·168 | 0·298 | 0·211 |
| 0·014 | 0·086 | 0·191 | 0·289 | 0·323 |

Table 11: The rotation matrix that relates the log quarterly price dividend ratios of the 25 Fama-French portfolios to their second principal component.

for the annual data and sometimes two for the quarterly data. The estimated innovations of the first and second principal component thus obtained are denoted by $IF_1^f$ and $IF_2^f$ respectively with $f = a, q$ representing the measurement frequency.

## 7.2 Relation between the principal components and dividend and consumption growth

I present evidence that $F_1$ is strongly related to consumption growth volatility and that $F_2$ predicts future (both market and cross-sectional) dividend and real time consumption growth. I also present indicative evidence that it is weakly related to future long term consumption growth calculated using revised data. I find that while the two principal components technically only identify a subspace, their relations to consumption growth and volatility are very different and that they seem to be naturally identifiable as affine functions of the underlying state variables $V$ and $X$ respectively.

I estimated the innovations of quarterly consumption growth $\epsilon_{v,t}$ as the OLS residuals obtained on regressing it on $n$ lags of itself, $n$ being the smallest value for which they were uncorrelated at the 10% level according to both the Ljung-Box and Durbin-Watson tests. $n$ was found to be three for this data series. Using these estimated innovations, I estimated the consumption volatility series as

$$v_t^n = \log \sum_{i=1}^{n} \frac{\epsilon^2_{v,\lfloor t+n/2-i \rfloor}}{n} \tag{21}$$

I summarize the results of regressing $v_t^{24}, v_t^{12}$ and $v_t^6$ on $F_1^q$ and $F_2^q$ in table 12. These results show that $F_1$ is very closely related to consumption growth volatility with the $R^2$ of the 24 quarter volatility being as high as 81%. The $R^2$ for 6 quarter volatility, where the measurement error is likely to be high, is still quite high at 47%. In contrast, the results show no evidence that $F_2$ is related to consumption growth volatility. Further, I note that the coefficient of $F_1$ in the regression is very similar for the volatility estimated over several horizons which indicates that the relation is robust.

**Regression of consumption growth volatility on $F_1^q$ and $F_2^q$**

| | $F_1^q$ | $F_2^q$ | $R^2$ |
|---|---|---|---|
| 24 quarter volatility | $-0.213^{***}$ (0.027) | 0.081 (0.094) | 81.2% |
| 12 quarter volatility | $-0.234^{***}$ (0.052) | 0.074 (0.190) | 62.7% |
| 6 quarter volatility | $-0.235^{***}$ (0.062) | 0.024 (0.209) | 46.9% |

Table 12: Results of regressing volatility as defined in (21) against $F_1^q$ and $F_2^q$. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in (Newey and West 1994).

I present the results of regressing annual real market dividend growth (i.e., growth of annual market dividends deflated by the CPI) on the lagged values of $F_1$ and $F_2$ in table 13. I find, from them, that $F_2$, but not $F_1$, predicts market dividend growth. I note the interesting fact that $F_2$, which weights the value portfolios more heavily, predicts future market dividend growth better than the log market price dividend ratio (whose inability to predict dividend growth is well known (Cochrane 2005)). I hypothesize that this is because value stocks have a low duration which makes their price dividend ratios depend more on dividend growth than on future expected excess returns. I also find that $F_2$ strongly predicts cross-sectional dividend growth and that the $F$ statistic for the equality of the regression coefficients obtained on regressing real dividend growth of each of the 25 portfolios on the lagged value of $F_2$ is 2.20. This value implies that such equality can be rejected at the 0.1% level. I find that the regression coefficients are higher for portfolios of value and small stocks. This is not surprising given the form of the rotation matrix relating $F_2$ to the log price dividend ratios.[20] I find that the corresponding $F$ statistic for $F_1$ is 1.73

[20] As in most studies of cross-sectional dividends such as (Bansal, Dittmar, and Lundblad 2005), most of the individual coefficients are not significant but they are significantly *different*

with $p = 0.015$. However, apart from larger coefficients for the portfolios corresponding to the smallest stocks, I find little cross-sectional variation among the regression coefficients obtained on regressing portfolio dividend growths against the lagged value of $F_1$. I used annual values for the above analysis in order to eliminate issues arising from dividend and CPI seasonality and to minimize the confounding effects that arise from overlapping regressions.

**Regression of market dividend growth on $F_1^a$ and $F_2^a$ and the log market price dividend ratio**

| | $F_1^a$ | $F_2^a$ | $\log(P/D)_m$ | $R^2$ |
|---|---|---|---|---|
| 1 yr. Market div. growth | 0.0004 (0.0031) | 0.0317*** (0.0079) | | 16.0% |
| 1 yr. Market div. growth | | | 0.003 (0.027) | 0.0% |
| 3 yr. Market div. growth | 0.0008 (0.0135) | 0.0571** (0.0261) | | 11.6% |
| 3 yr. Market div. growth | | | 0.005 (0.107) | 0.0% |

Table 13: Results of regressing real annual market dividend growth against lagged $F_1^a$ and $F_2^a$. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in (Newey and West 1994). The regressions using the log market price dividend ratio are from 1944 to be consistent with the others.

I present the results of regressing annual real time consumption growth against the lagged values of $F_1^a$ and $F_2^a$ in table 14. I find, from them, that $F_2^a$ also predicts real time consumption growth as defined in the data section. This is in accordance with the long run risk hypothesis that dividend and consumption growth share the same persistent component. I note that while the use of this measure of consumption is not standard, it is more relevant for the current analysis as it better matches the information structure of the consumers in the economy. (It is also possible that real time data captures the sentiment of consumers as it reflects their current view of the state of the economy.)

I find no strong relation between the two principal components and short term consumption growth when consumption is defined as the consumption of nondurables and services estimated with the use of current data. However, I also find, from the results of regressing current annual consumption growth against the fourth lag of $IF_2^a$, which I summarize in table 15, that there is a significant negative relation between them which could mask the true long run relation between $F_2^a$ and future consumption growth. Keeping this in mind, I use the results of regressing five year consumption growth after the first five years (i.e. $c_{t+10} - c_{t+5}$) against the lagged principal components, which I summarize in table 16, and the fact that the coefficients therein are significant at the 10% level (using Newey-West corrected standard errors), to show that there is some evidence that $F_2$ is positively related to future long term consumption growth. I find that this positive relationship is concentrated in the services sector and that regressing three or five year services consumption growth after the first five years on the lagged principal components gives rise to coefficients

from each other.

which are significant at the 1% level. This result is consistent with the evidence documented by Marakani (2009a) that services consumption growth exhibits highly significant long term autocorrelations unlike nondurables consumption growth. More importantly, I find that the significant coefficient is that of $F_2$ and that it is of the same sign as it's regression coefficient for future market and cross-sectional dividend growth as expected from the long run risk model. I note that the log market price dividend ratio does not predict consumption growth even after a gap of a few years unlike $F_2$ and that the $R^2$ of the regression of $c_{t+10} - c_{t+5}$ on the lagged log market price dividend ratio is a negligible $10^{-6}$.[21]

**Regression of real time annual consumption growth on lagged values of $F_1^a$ and $F_2^a$.**

| | $F_1^a$ | $F_2^a$ | $R^2$ |
|---|---|---|---|
| $\Delta c_{t+1}^{RT}$ | $-2 \times 10^{-4}(9 \times 10^{-4})$ | 0.0068 (0.0037) | 17.4% |
| $\Delta c_{t+1}^{RT} + \Delta c_{t+2}^{RT}$ | $-5 \times 10^{-4}(1.5 \times 10^{-3})$ | $0.0123^*$ (0.0050) | 18.9% |

Table 14: Results of regressing real time annual consumption growth on lagged values of $F_1^a$ and $F_2^a$. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in (Newey and West 1994).

In balance, I find strong evidence that $F_2$ predicts real time consumption growth as well as market and cross sectional dividend growth and some indicative evidence that it is related to long term consumption growth measured in the conventional manner. Hence, it is reasonable to consider it as an affine function of the component $X$ (since I find only one factor related to these quantities, I can drop the now superfluous subscript). I also find strong evidence that $F_1$ is related to contemporaneous consumption growth volatility and it is thus reasonable to consider it as an affine function of $V$ (again, since I only find one such factor, I drop the now superfluous subscript).

While I am of the opinion that real time data provides a better measure of consumption growth in the context of long run risk models, some readers will be concerned at the lack of evidence for a strong relationship between conventionally measured consumption growth and log price dividend ratios. I note that the lack of this evidence is not surprising as consumption growth is known to be largely unforecastable in the post WW2 period. In order to address these concerns, I note that the relationship I find using conventionally measured consumption growth, while weak, is much stronger than that obtained when only the log market price dividend ratio is used as by Beeler and Campbell (2009). I also note that prior studies examining the relation between returns or dividends and future consumption growth conclude that such relations exist even when the regression coefficients are not conventionally significant. For example, I note that the regressions that provide evidence for the relation between SMB, HML and future consumption growth in (Parker and Julliard 2005) and for the relation between cross sectional dividend growth and consumption growth in

[21] Using services consumption growth instead increases this $R^2$ to 1.4%.

(Bansal, Dittmar, and Kiku 2009) do not give rise to statistically significant coefficients. Further, I note that, as pointed out by Hansen and Sargent (2007), the predictable component of consumption growth can be small enough to be undetectable by standard statistical tests but still large enough to be economically important and the weak but suggestive relation I find is strong enough to significantly affect asset prices.[22]

| **Regression of annual consumption growth on the fourth lag of $IF_2^a$** | | | |
|---|---|---|---|
| | Intercept | $IF_{2,t-3}^a$ | $R^2$ |
| $\Delta c_{t+1}$ | 0.0194 (0.0015) | $-7.90 \times 10^{-3}$ ($3.67 \times 10^{-3}$) | 7.3% |

Table 15: Regression of annual consumption growth on the fourth lag of $IF_2^a$.

| **Regression of five year consumption growth after five years on lagged $F_1^a$ and $F_2^a$** | | | |
|---|---|---|---|
| Intercept | $F_1^a$ | $F_2^a$ | $R^2$ |
| 0.104 (0.010) | -0.0015 (0.0039) | 0.0164 (0.0090) | 19.1% |
| **Regression of five year services consumption growth after five years on lagged $F_1^a$ and $F_2^a$** | | | |
| Intercept | $F_1^a$ | $F_2^a$ | $R^2$ |
| 0.121 (0.011) | -0.0048 (0.0049) | 0.0214** (0.0066) | 46.4% |

Table 16: Results of regressing five year overall and services consumption growth after five years (i.e., $c_{t+10} - c_{t+5}$) on lagged $F_1^a$ and $F_2^a$. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in (Newey and West 1994).

I investigate the relation between the innovations of the principal components and the Fama-French factors as they have been proposed as proxies for future consumption growth (by Parker and Julliard (2005)) and consumption growth volatility (by Boguth and Kuehn (2008)). I summarize the results of regressing $IF_1^a$ and $IF_2^a$ on the annual Fama-French factors in table 17. I find, from them, that $IF_1^a$ and $IF_2^a$ can be approximately written as $Mkt + SMB$ and $Mkt + HML$ respectively. In other words, I find that in the framework of this analysis, excess market returns are related to both consumption growth and consumption growth volatility, that SMB is related to consumption growth volatility and that HML is related to future consumption and dividend growth.

## 7.3 Asset pricing tests

In the asset pricing tests below, I ignore the contemporaneous consumption growth factor which is technically required for completeness due to the presence of the $W_{t+\Delta t} - W_t$ term in (9). I do so as previous studies have shown that

[22]This is shown in unreported results of asset pricing tests using the innovations of projections of future consumption growth on $F_1$ and $F_2$.

**Regressions of $IF_1^a$ and $IF_2^a$ on the Fama-French factors**

| | Intercept | $R_m - R_f$ | $SMB$ | $HML$ | $R^2$ |
|---|---|---|---|---|---|
| $IF_1^a$ | $-0.39^{***}$ (0.09) | $3.95^{***}$ (0.42) | $2.10^{***}$ (0.58) | 0.50 (0.56) | 68.8% |
| $IF_2^a$ | $-0.19^{***}$ (0.04) | $1.42^{***}$ (0.18) | $-0.22$ (0.25) | $1.42^{***}$ (0.25) | 60.0% |

Table 17: Results of regressing $IF_1^a$ and $IF_2^a$ on the annual Fama-French factors.

contemporaneous consumption growth is incapable of explaining excess asset returns. A similar procedure is followed by Malloy, Moskowitz, and Vissing-Jørgensen (2009).[23]

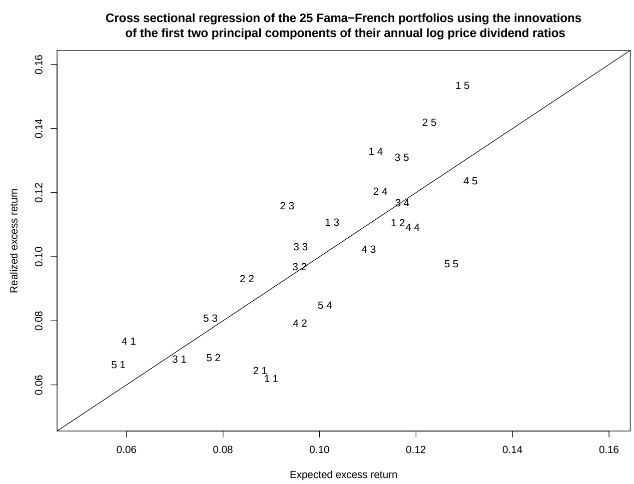


Figure 10: Results of the cross sectional regression of the 25 Fama-French portfolios using $IF_1^a$ and $IF_2^a$.

Empirical analysis of the beta pricing relationship (17) (ignoring the first term in it, as is conventional in the literature) reveals that it's performance was good at the annual time scale but less impressive at the quarterly time scale. I summarize the results of the OLS and WLS cross sectional regressions for this beta pricing relationship (using $IF_1$ and $IF_2$) at the annual and quarterly frequency in tables 18 and 19 respectively. From these results, I find that the model does not generate a significantly non-zero beta rate, denoted by $\gamma_0$, unlike the Fama-French three factor model.[24] I find, from figure 10, that the cross sectional performance of the model at the annual frequency is fairly good with the $R^2$ of the cross sectional regression being 64.8%. However, I also find, from figure 11, that the cross sectional performance is only average at the

[23]In unreported results, I find that the results are not significantly different when the contemporaneous consumption growth factor is included.

[24]The same is true for innovations of the principal components at the monthly frequency.

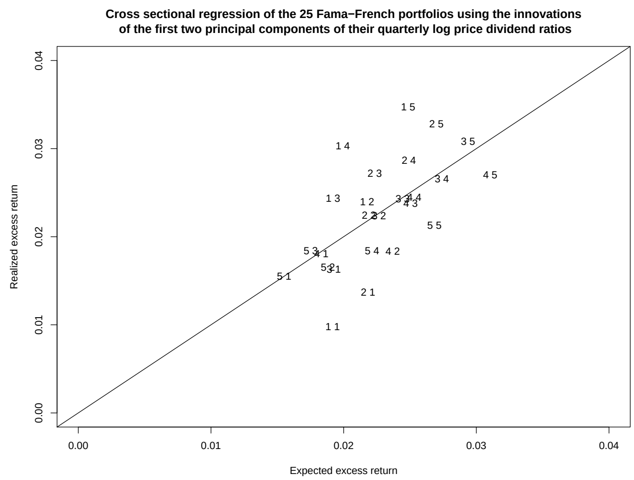


Figure 11: Results of the cross sectional regression of the 25 Fama-French portfolios using $IF_1^q$ and $IF_2^q$.

quarterly frequency with the $R^2$ of the cross sectional regression being 38.4%.[25] I find that $\gamma_{IF_1}$ and $\gamma_{IF_2}$, the coefficients of $\beta_{IF_1}$ and $\beta_{IF_2}$ in the cross sectional regression, are jointly significantly different from zero at the 5% level at the annual frequency and at the 1% level at the quarterly frequency (as well as at the monthly frequency). A common concern when using the cross sectional regression methodology is that the betas do not show sufficient cross sectional variation. However, from the results in tables 20 and 21, I find that this is not the case here and that the $F$ test strongly rejects the hypothesis that the betas are equal (for either $IF_1$ or $IF_2$) for all the portfolios with $p < 10^{-4}$. I tabulate the pricing errors generated by the two cross sectional regressions in tables 22 and 23 respectively.

Possible reasons for the poor performance of the model at the quarterly frequency include the pronounced seasonal nature of dividends and the short term effects of announcements regarding future dividends, both of which might not be properly captured by price dividend ratios at this frequency. Another likely possibility is that there are short term components, like the one proposed by Zhou and Zhu (2009), which are not revealed by the factor analysis. However, in my opinion, the most likely reason, given the results of the analysis with the beta pricing relationship (17) which are summarized below, is that it is the result of ignoring the second term in (13).

Since I estimate the principal components from the 25 Fama-French portfolios, the reader might argue that the above results just reflect the well known

[25] At the monthly frequency, the $R^2$ reduces to 33.4% but the zero beta rate is still not significantly different from zero.

**Results of the annual cross sectional regression on the basis of (17)**

| | Intercept | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|
| OLS | -0.026 | 0.498 | 0.432 | 64.8% |
| | (-0.77) | (2.17) | (3.77) | (61.6%) |
| | (-0.52) | (1.60) | (2.67) | |
| WLS | -0.032 | 0.586 | 0.407 | |
| | (-0.67) | (1.92) | (2.64) | |

Table 18: Results of the two pass cross sectional regression of the 25 Fama-French portfolios using $IF_1^a$ and $IF_2^a$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

**Results of the quarterly cross sectional regression on the basis of (17)**

| | Intercept | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|
| OLS | -0.010 | 0.139 | 0.110 | 38.4% |
| | (-1.08) | (2.50) | (3.65) | (32.9%) |
| | (-0.91) | (2.21) | (3.13) | |
| WLS | -0.011 | 0.151 | 0.101 | |
| | (-1.04) | (2.46) | (3.08) | |

Table 19: Results of the two pass cross sectional regression of the 25 Fama-French portfolios using $IF_1^q$ and $IF_2^q$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

strong factor structure of their returns. Hence, I check that other portfolios are also priced using these innovations. For this purpose, I use ten portfolios each formed on the basis of long term reversal, short term reversal and price earnings ratio as described in the data section.

I find that these thirty portfolios are well priced by $IF_1^a$ and $IF_2^a$. The zero beta rate is negligible (-0.003) and the two innovations are jointly significantly priced at the 2% level. The $R^2$ of the second pass regression is 75%. I summarize these results in table 24 and graph the cross sectional regression in figure 12. I summarize the corresponding results when the innovations of the quarterly log price dividend ratios are used to price the thirty portfolios in table 25. I find that the zero beta rate is lower at the quarterly frequency but is still not significantly

**Dispersion in the betas corresponding to $IF_1^a$**

Beta

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | 0.226 (0.033) | 0.211 (0.023) | 0.169 (0.021) | 0.155 (0.018) | 0.159 (0.022) |
| 2 | 0.190 (0.025) | 0.150 (0.018) | 0.144 (0.017) | 0.141 (0.015) | 0.124 (0.020) |
| 3 | 0.167 (0.023) | 0.130 (0.016) | 0.109 (0.015) | 0.119 (0.016) | 0.113 (0.018) |
| 4 | 0.144 (0.020) | 0.107 (0.016) | 0.111 (0.015) | 0.110 (0.015) | 0.134 (0.020) |
| 5 (large) | 0.112 (0.019) | 0.084 (0.014) | 0.076 (0.017) | 0.080 (0.017) | 0.089 (0.019) |

$F$-stat $= 5.55$ ($p < 10^{-16}$)

**Dispersion in the betas corresponding to $IF_2^a$**

Beta

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | 0.007 (0.087) | 0.085 (0.060) | 0.102 (0.053) | 0.139 (0.048) | 0.176 (0.058) |
| 2 | 0.044 (0.065) | 0.084 (0.047) | 0.109 (0.044) | 0.158 (0.040) | 0.201 (0.051) |
| 3 | 0.031 (0.058) | 0.131 (0.041) | 0.156 (0.039) | 0.193 (0.042) | 0.200 (0.047) |
| 4 | 0.033 (0.051) | 0.159 (0.040) | 0.186 (0.042) | 0.208 (0.040) | 0.209 (0.051) |
| 5 (large) | 0.065 (0.048) | 0.143 (0.037) | 0.151 (0.044) | 0.202 (0.045) | 0.251 (0.049) |

$F$-stat $= 2.18$ ($p = 8.1 \times 10^{-4}$)

Table 20: Dispersion in the betas of the continuously compounded excess returns of the 25 Fama French portfolios on $IF_1^a$ and $IF_2^a$. The standard errors are reported in brackets and the $F$ statistic corresponds to the test that the (individual) betas are identical for all the portfolios.

**Dispersion in the betas corresponding to $IF_1^q$**

Beta

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | 0.277 (0.011) | 0.237 (0.008) | 0.204 (0.007) | 0.192 (0.006) | 0.199 (0.008) |
| 2 | 0.244 (0.008) | 0.203 (0.006) | 0.176 (0.006) | 0.168 (0.005) | 0.177 (0.007) |
| 3 | 0.219 (0.008) | 0.177 (0.005) | 0.155 (0.005) | 0.145 (0.005) | 0.155 (0.007) |
| 4 | 0.193 (0.007) | 0.159 (0.005) | 0.142 (0.005) | 0.139 (0.005) | 0.153 (0.007) |
| 5 (large) | 0.138 (0.007) | 0.117 (0.006) | 0.098 (0.006) | 0.102 (0.006) | 0.108 (0.007) |

$F$-stat $= 36.64$ ($p < 10^{-16}$)

**Dispersion in the betas corresponding to $IF_2^q$**

Beta

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | −0.084 (0.030) | −0.008 (0.022) | 0.011 (0.019) | 0.032 (0.018) | 0.068 (0.023) |
| 2 | −0.017 (0.024) | 0.036 (0.018) | 0.074 (0.016) | 0.108 (0.016) | 0.115 (0.021) |
| 3 | −0.008 (0.023) | 0.076 (0.015) | 0.121 (0.014) | 0.160 (0.015) | 0.165 (0.019) |
| 4 | 0.016 (0.021) | 0.108 (0.015) | 0.142 (0.015) | 0.148 (0.015) | 0.184 (0.020) |
| 5 (large) | 0.061 (0.020) | 0.117 (0.016) | 0.130 (0.017) | 0.167 (0.017) | 0.201 (0.021) |

$F$-stat $= 2.69$ ($p = 1.5 \times 10^{-5}$)

Table 21: Dispersion in the betas of the continuously compounded excess returns of the 25 Fama French portfolios on $IF_1^q$ and $IF_2^q$. The standard errors are reported in brackets and the $F$ statistic corresponds to the test that the (individual) betas are identical for all the portfolios.

**Pricing errors ×100 produced by the annual cross sectional regression using $IF_1^a$ and $IF_2^a$**

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | 2·80 | 0·56 | −0·81 | −2·13 | −2·38 |
| 2 | 2·32 | −0·82 | −2·25 | −0·78 | −1·92 |
| 3 | 0·29 | −0·10 | −0·71 | 0·03 | −1·40 |
| 4 | −1·33 | 1·66 | 0·78 | 1·01 | 0·76 |
| 5 (large) | −0·81 | 0·96 | −0·34 | 1·63 | 2·95 |

Table 22: Pricing errors (in %) produced by the annual cross sectional regression of the 25 Fama-French portfolios on $IF_1^a$ and $IF_2^a$.

**Pricing errors ×100 produced by the quarterly cross sectional regression using $IF_1^q$ and $IF_2^q$**

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | 0·94 | −0·22 | −0·52 | −1·04 | −0·99 |
| 2 | 0·81 | −0·06 | −0·48 | −0·37 | −0·58 |
| 3 | 0·29 | 0·03 | 0·02 | 0·08 | −0·15 |
| 4 | 0·02 | 0·53 | 0·12 | 0·09 | 0·40 |
| 5 (large) | 0·00 | 0·23 | −0·09 | 0·37 | 0·55 |

Table 23: Pricing errors (in %) produced by the quarterly cross sectional regression of the 25 Fama-French portfolios on $IF_1^q$ and $IF_2^q$.

different from zero and that the two innovations are jointly significantly priced at the 1% level. The $R^2$ of the cross sectional regression is 72%. I tabulate the pricing errors generated by these cross sectional regressions in tables 26 and 27.

Hence, I conclude that the innovations of the first two principal components of the log price dividend ratios of the 25 Fama-French portfolios are able to price a variety of equity portfolios which have posed a challenge to traditional asset pricing models. This is particularly true at the annual time scale.

The above results show that the pricing relationship (17) works well at the annual frequency but not very well at the quarterly frequency. The pricing relationship (13), which does not induce biases into estimates of the market prices of risk, however, works much well at both frequencies.[26] I tabulate the coefficients obtained from the annual and quarterly cross sectional regressions in tables 28 and 29 respectively. The $R^2$ for the cross sectional regressions at the annual and quarterly frequencies which are graphed in figures 14 and 15 are 77.8% and 69.5% respectively. The results indicate that once the lagged values of the principal components are included, the coefficients of the lagged values and innovations of the first principal component become significant while

[26] Since this pricing relationship is not in terms of real and not excess returns, the CPI was used to convert nominal returns to real.

**Results of the annual cross sectional regression with 30 portfolios on the basis of (17)**

| | Intercept | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|
| OLS | -0.007 | 0.292 | 0.419 | 72.7% |
| | (-0.31) | (1.40) | (4.21) | (70.7%) |
| | (-0.21) | (1.07) | (3.13) | |
| WLS | -0.016 | 0.375 | 0.415 | |
| | (-0.47) | (1.48) | (3.32) | |

Table 24: Results of the two pass cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using $IF_1^a$ and $IF_2^a$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

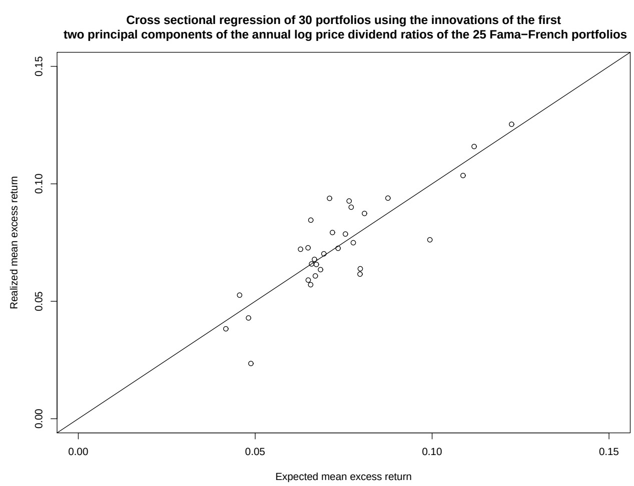


Figure 12: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using $IF_1^a$ and $IF_2^a$.

**Results of the quarterly cross sectional regression with 30 portfolios on the basis of (17)**

| | Intercept | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|
| OLS | -0.023 | 0.166 | 0.172 | 63.9% |
| | (-2.59) | (3.07) | (4.98) | (61.2%) |
| | (-1.85) | (2.43) | (3.66) | |
| WLS | -0.022 | 0.164 | 0.165 | |
| | (-1.88) | (2.50) | (3.78) | |

Table 25: Results of the two pass cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using $IF_1^q$ and $IF_2^q$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

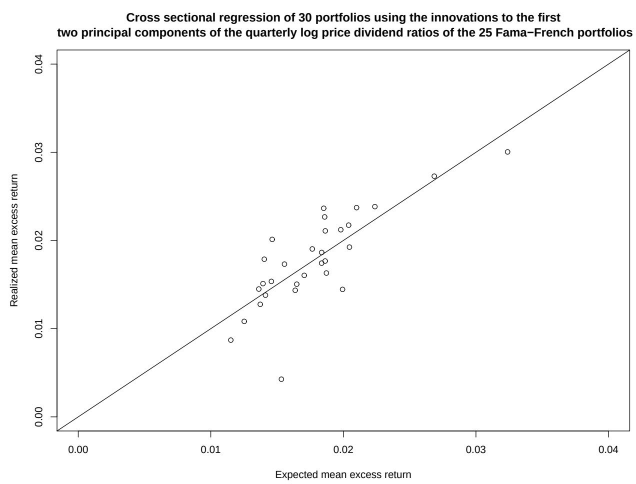


Figure 13: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using $IF_1^q$ and $IF_2^q$.

**Pricing errors ×100 produced by the annual cross sectional regression with 30 portfolios using $IF_1^a$ and $IF_2^a$**

| | bottom 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | top 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Long term reversal | −0·64 | −1·88 | −0·74 | 0·09 | 0·28 | −0·93 | −0·78 | −0·11 | 1·81 | 0·16 |
| Short term reversal | 2·32 | −2·28 | −1·61 | −0·31 | −0·08 | 1·58 | 0·86 | 0·50 | 0·34 | 2·53 |
| E/P ratio | −0·70 | 0·52 | 0·62 | 0·60 | 0·00 | −0·65 | −1·29 | 0·52 | −0·40 | −0·29 |

Table 26: Pricing errors (in %) produced by the annual cross sectional regression of three sets of ten portfolios formed on the basis of long term reversal, short term reversal and earnings to price ratio on $IF_1^a$ and $IF_2^a$.

**Pricing errors ×100 produced by the quarterly cross sectional regression with 30 portfolios using $IF_1^q$ and $IF_2^q$**

| | bottom 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | top 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Long term reversal | −0·27 | −0·25 | −0·55 | 0·09 | −0·03 | −0·38 | 0·10 | 0·24 | 0·55 | 0·14 |
| Short term reversal | 0·12 | −0·51 | −0·41 | −0·14 | −0·18 | −0·08 | 0·03 | −0·09 | 0·28 | 1·11 |
| E/P ratio | 0·10 | 0·17 | −0·12 | 0·20 | 0·10 | −0·14 | −0·13 | −0·15 | −0·04 | 0·23 |

Table 27: Pricing errors (in %) produced by the quarterly cross sectional regression of three sets of ten portfolios formed on the basis of long term reversal, short term reversal and earnings to price ratio on $IF_1^q$ and $IF_2^q$.

those of the second principal component become less significant (they are still, however, significant at the 5% level). Since the lagged values of the principal components correspond to the expected excess returns, which in long risk models, are determined by consumption growth volatility, this indicates that once these are taken into account, consumption volatility risk is of greater importance than consumption growth risk, a result consistent with the findings of Boguth and Kuehn (2008). The corresponding results for the three sets of ten portfolios based on long term reversal, short term reversal and earnings to price ratio are summarized in tables 32 and 33 and the cross sectional regressions for these portfolios are graphed in figures 16 and 17 respectively. The pricing errors generated by the cross sectional regressions are tabulated in tables 34 and 35. The results for these out of sample regressions indicate that the lagged values of the principal components, unlike their innovations, might not contain global pricing information applicable to all portfolios as is implied by the long run risk model and that the information they carry about the excess returns might only be applicable to the set of portfolios from which they are formed. Further, the fact that the $F$ test does not show significant dispersion of betas for the lagged values of the principal components at the annual and quarterly frequencies means that the effects noted here are not robustly established, particularly since betas of persistent factors cannot be precisely estimated with a short data sample. Further research, is thus required to establish the true importance of varying expected excess returns in such asset pricing tests and whether they are indeed determined by global quantities as is implied by long run risk models.

I note that the identification of the two factors in this study also enables the determination of the relative importance of cash flow and discount rate risks for cross-sectional returns in the context of long run risk models. This is because the rate at which future equity cash flows are discounted is largely determined by the consumption volatility in these models, which in turn means that the first factor proxies for discount rate risk and that the second proxies for cash flow risk. The results of the analysis using the innovations of the two factors indicate that cash flow risk is cross-sectionally more important than discount rate risk. However, this result is reversed when the lagged factors are included in the estimation. Given the evidence of lack of global information in the lagged factors and the fact that betas on them do not show significant cross-sectional variation, I put more confidence in the result without them. This result underlines the importance of cash flow risk and contributes to the recent strand of literature that demonstrates that cash flow risk can explain a large proportion of the cross-sectional return variation (Campbell and Vuolteenaho 2004) (Bansal, Dittmar, and Lundblad 2005) (Cohen, Polk, and Vuolteenaho 2008) (Campbell, Polk, and Vuolteenaho 2009) (Da and Warachka 2009).

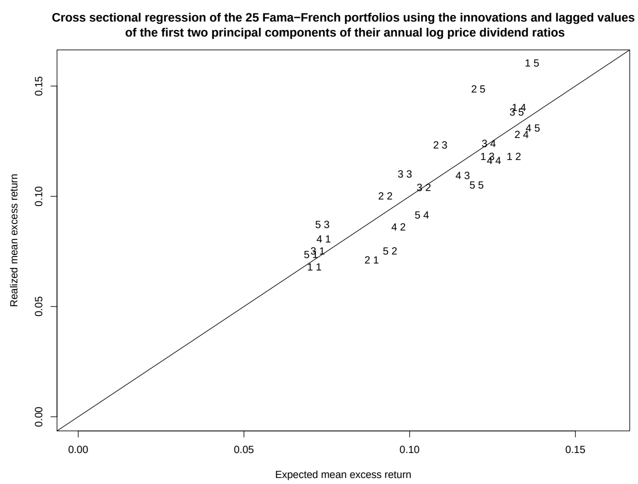


Figure 14: Results of the cross sectional regression of the 25 Fama-French portfolios using $IF_1^a$, $IF_2^a$ and lagged values of $F_1^a$ and $F_2^a$.

Since it is largely $F_2$ that predicts future dividend and consumption growth, a rough estimate of the risk aversion (or, more accurately, $\gamma - 1/\psi \approx \gamma$, since $\psi > 1$) implied by the model can be made using the cross sectional regression coefficient for $IF_2$. To simplify the notation in the analysis below, I suppress the superfluous index for the only identified $X$ component. I first note the relation[27] (which follows from the identification $F_2 \propto X$ and the fact that there is only

[27] This is exact only when $IF_2$ alone is used in the cross sectional regression. I find that performing the cross sectional regression using $IF_2$ alone produces essentially the same result.

**Results of annual cross sectional regressions on the basis of (13)**

| | Intercept | $\gamma_{F_1}$ | $\gamma_{F_2}$ | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|---|---|
| OLS | -0.012 | 1.39 | -0.587 | 0.701 | 0.304 | 77.8% |
| | (-0.35) | (1.75) | (-3.15) | (2.88) | (2.69) | (73.3%) |
| | (-0.23) | (1.23) | (-2.22) | (2.04) | (1.85) | |
| WLS | -0.018 | 1.11 | -0.526 | 0.705 | 0.307 | |
| | (-0.36) | (1.04) | (-2.18) | (2.13) | (1.99) | |

Table 28: Results of the two pass cross sectional regression of the 25 Fama-French portfolios using lagged $F_1^a$ and $F_2^a$ as well as $IF_1^a$ and $IF_2^a$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

**Results of quarterly cross sectional regressions on the basis of (13)**

| | Intercept | $\lambda_{F_1}$ | $\gamma_{F_2}$ | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|---|---|
| OLS | -0.002 | 2.63 | -0.552 | 0.179 | 0.073 | 69.5% |
| | (-0.22) | (3.56) | (-2.74) | (2.98) | (2.42) | (63.4%) |
| | (-0.14) | (2.33) | (-1.79) | (2.08) | (1.62) | |
| WLS | -0.002 | 2.05 | -0.602 | 0.171 | 0.063 | |
| | (-0.14) | (2.16) | (-2.14) | (2.16) | (1.64) | |

Table 29: Results of the two pass cross sectional regression of the 25 Fama-French portfolios using lagged $F_1^q$ and $F_2^q$ as well as $IF_1^q$ and $IF_2^q$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

**Pricing errors $\times 100$ produced by the annual cross sectional regression using $F_1^a$, $F_2^a$, $IF_1^a$ and $IF_2^a$**

| | growth 1 | 2 | 3 | 4 | value 5 |
|---|---|---|---|---|---|
| 1 (small) | −0·25 | 0·75 | −0·04 | −1·32 | −2·94 |
| 2 | 1·16 | −1·34 | −1·98 | −0·02 | −3·39 |
| 3 | −0·89 | −0·56 | −1·75 | −0·59 | −1·17 |
| 4 | −1·25 | 0·48 | 0·08 | 0·33 | 0·05 |
| 5 (large) | −0·93 | 1·29 | −1·94 | 0·64 | 0·93 |

Table 30: Pricing errors (in %) produced by the annual cross sectional regression of the 25 Fama-French portfolios on lagged values of $F_1^a$ and $F_2^a$ and concurrent values of $IF_1^a$ and $IF_2^a$.

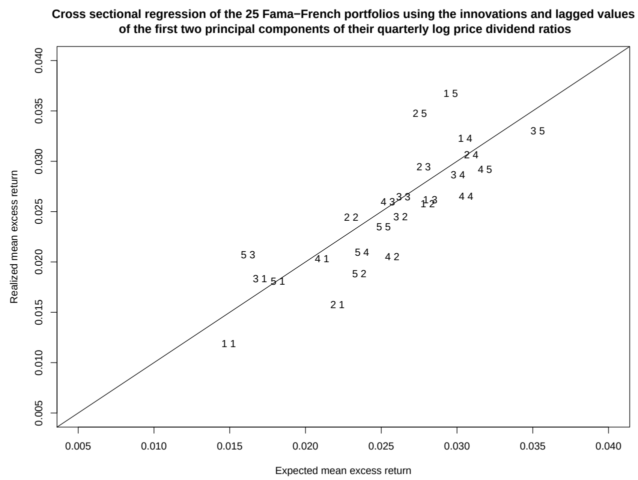


Figure 15: Results of the cross sectional regression of the 25 Fama-French portfolios using $IF_1^q$, $IF_2^q$ and lagged values of $F_1^q$ and $F_2^q$.

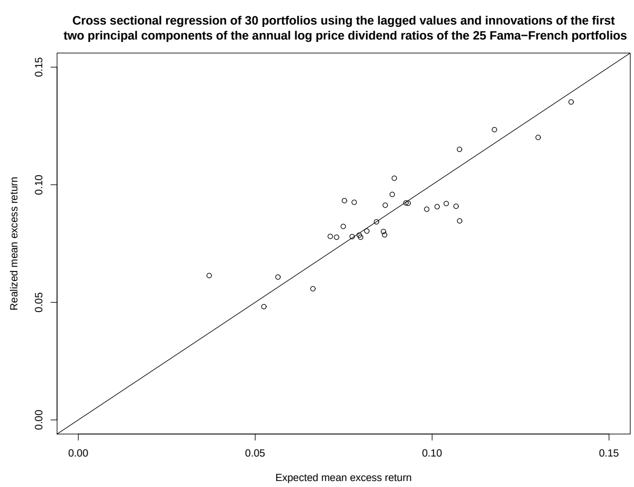


Figure 16: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using lagged $F_1^a$ and $F_2^a$ as well as $IF_1^a$ and $IF_2^a$.

**Pricing errors ×100 produced by the quarterly cross sectional regression using $F_1^q$, $F_2^q$, $IF_1^q$ and $IF_2^q$**

| | growth | | | | value |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| 1 (small) | 0·07 | 0·00 | −0·03 | −0·41 | −0·95 |
| 2 | 0·40 | −0·38 | −0·40 | −0·20 | −0·96 |
| 3 | −0·37 | −0·05 | −0·24 | −0·09 | −0·01 |
| 4 | −0·16 | 0·28 | −0·29 | 0·17 | 0·03 |
| 5 (large) | −0·23 | 0·24 | −0·68 | 0·04 | −0·07 |

Table 31: Pricing errors (in %) produced by the quarterly cross sectional regression of the 25 Fama-French portfolios on lagged values of $F_1^q$ and $F_2^q$ and concurrent values of $IF_1^q$ and $IF_2^q$.

**Results of annual cross sectional regressions for 30 portfolios on the basis of (13)**

| | Intercept | $\gamma_{F_1}$ | $\gamma_{F_2}$ | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|---|---|
| OLS | -0.012 | 1.22 | -0.037 | 0.389 | 0.450 | 77.24% |
| | (-0.44) | (1.75) | (-0.14) | (1.73) | (4.74) | (73.6%) |
| | (-0.28) | (1.25) | (-0.09) | (1.24) | (3.38) | |
| WLS | -0.019 | 1.01 | -0.060 | 0.443 | 0.445 | |
| | (-0.48) | (1.07) | (-0.16) | (1.48) | (3.36) | |

Table 32: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using lagged $F_1^a$ and $F_2^a$ as well as $IF_1^a$ and $IF_2^a$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

**Results of quarterly cross sectional regressions for 30 portfolios on the basis of (13)**

| | Intercept | $\gamma_{F_1}$ | $\gamma_{F_2}$ | $\gamma_{IF_1}$ | $\gamma_{IF_2}$ | $R^2$ |
|---|---|---|---|---|---|---|
| OLS | -0.027 | 0.493 | -0.426 | 0.211 | 0.173 | 72.3% |
| | (-2.61) | (0.82) | (-1.66) | (3.22) | (5.05) | (67.9%) |
| | (-1.72) | (0.55) | (-1.11) | (2.29) | (3.44) | |
| WLS | -0.026 | 0.497 | -0.366 | 0.207 | 0.171 | |
| | (-1.75) | (0.59) | (-1.07) | (2.36) | (3.55) | |

Table 33: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using lagged $F_1^q$ and $F_2^q$ as well as $IF_1^q$ and $IF_2^q$. For the OLS coefficients, the $t$ values without and with the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the $t$ value with the correction is reported. The $R^2$ adjusted for the number of variables is reported below the unadjusted $R^2$.

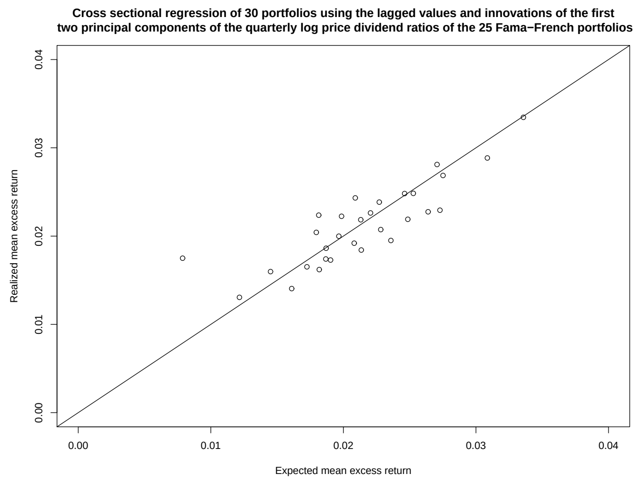


Figure 17: Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio) using lagged $F_1^q$ and $F_2^q$ as well as $IF_1^q$ and $IF_2^q$.

**Pricing errors $\times 100$ produced by the annual cross sectional regression with 30 portfolios using $F_1^a$, $F_2^a$, $IF_1^a$ and $IF_2^a$**

| | bottom 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | top 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Long term reversal | 1·35 | −2·31 | −1·59 | −0·03 | 0·00 | 1·46 | 0·68 | 0·05 | −0·43 | 2·44 |
| Short term reversal | 0·73 | −0·89 | −0·11 | 0·45 | 0·72 | −0·61 | −0·79 | −0·12 | 1·80 | −0·08 |
| E/P ratio | −1·05 | 0·43 | 0·74 | 0·47 | −0·21 | −1·08 | −1·20 | 0·58 | −0·98 | −0·41 |

Table 34: Pricing errors (in %) produced by the annual cross sectional regression of three sets of ten portfolios formed on the basis of long term reversal, short term reversal and earnings to price ratio on lagged values of $F_1^a$ and $F_2^a$ and concurrent values of $IF_1^a$ and $IF_2^a$.

**Pricing errors $\times 100$ produced by the quarterly cross sectional regression with 30 portfolios using $F_1^q$, $F_2^q$, $IF_1^q$ and $IF_2^q$**

| | bottom 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | top 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Long term reversal | 0·11 | −0·44 | −0·36 | −0·21 | −0·16 | −0·17 | −0·07 | −0·20 | 0·09 | 0·96 |
| Short term reversal | −0·20 | 0·15 | −0·01 | 0·25 | 0·03 | −0·30 | −0·04 | −0·06 | −0·20 | −0·01 |
| E/P ratio | 0·10 | 0·02 | −0·41 | 0·34 | 0·06 | −0·29 | 0·05 | 0·24 | 0·42 | −0·13 |

Table 35: Pricing errors (in %) produced by the quarterly cross sectional regression of three sets of ten portfolios formed on the basis of long term reversal, short term reversal and earnings to price ratio on lagged values of $F_1^q$ and $F_2^q$ and concurrent values of $IF_1^q$ and $IF_2^q$.

one identified $X$ component in the data used for this study)

$$\begin{aligned}\lambda_{IF_2} &= \frac{\gamma - 1/\psi}{1-\nu_1(1-\alpha\Delta t)}\text{Var}(IF_2)\left(\frac{IX}{IF_2}\right)\\ &= \frac{\gamma - 1/\psi}{1-\nu_1(1-\alpha\Delta t)}\text{Var}(IF_2)\left(\frac{X}{F_2}\right)\end{aligned} \tag{22}$$

where $IF_2$ and $IX$ stand for the innovations of the second principal component and $X$ respectively (which are taken to be proportional here). An estimate for $\frac{X}{F_2}$ can be obtained from the results of the regression in table 16. Using the relation (1), it is not difficult to see that coefficient obtained when regressing $c_{t+10} - c_{t+5}$ on $X$ is given by $\sum_{i=5}^{9}(\alpha\Delta t)^i$. Hence, the coefficient obtained when regressing on $F_2$ is

$$\sum_{i=5}^{9}(\alpha\Delta t)^i \frac{F_2}{X} \tag{23}$$

The persistence of the $X$ process is the same as that of $F_2$ since they are proportional and I find that it is about 0.85 on the annual time scale. Using this value and the regression coefficient of 0.0164 obtained in table 16, I obtain $\frac{F_2}{X} \approx 103.3$. This value, together with the annual value of $0.997^{12} \approx 0.97$ for $\nu_1$ estimated by Bansal and Yaron (2004), I obtain an estimate of 50.8 for $\gamma - 1/\psi$ or about 50 for $\gamma$. The analogous calculation with two year real time consumption growth gives a risk aversion estimate of about 70.[28] While high, this estimate is similar to the value of 60 obtained by Ludvigson, Chen, and Favilukis (2007). The leverage of market dividend growth on long term consumption growth can be similarly estimated from the results in table 13. It is found to be about 3.3, a value remarkably similar to that proposed by Bansal and Yaron (2004). Similar results are obtained when the Euler equation restrictions (11 together with 9) are examined using the generalized method of moments. This examination also shows that the model cannot be rejected at the 5% level at both the annual and quarterly frequencies.

The main reason that the risk aversion estimate I obtain is much higher than that proposed by Bansal and Yaron (2004) is that the volatility of consumption growth after the structural break ($1.85 \times 10^{-4}$) is much lower than that for the entire period for which data is available ($4.92 \times 10^{-4}$). If I scale the relative risk aversion value that I have obtained by the ratio of these volatilities, it's scaled value of 19 is very similar to the value of 16 obtained by Bansal, Yaron, and Kiku (2007). Hence, it is possible that a long run risk model which accounts for structural breaks or regime shifts in the parameters will require a much lower relative risk aversion to explain asset prices as the volatility of consumption growth will be much higher over the entire period.

[28] In unreported results, similar estimates for the risk aversion were obtained when using the innovations of the projection of future consumption growth on the two principal components of the log price dividend ratios, a methodology closer to the approach in (Ferson, Nallareddy, and Xie 2009) and (Bansal, Yaron, and Kiku 2007).

# 8 The relation between consumption growth and future excess market returns

I confirm the implication of the long run risk model and the observed large and positive correlation between $IF_1$ and $IF_2$ of 0.51 that consumption growth negatively predicts future excess market returns. The implication follows from the fact that the positive correlation between $IF_1$ and $IF_2$ is equivalent to a negative correlation between innovations to the predictable component of consumption growth and innovations to consumption growth volatility and that excess market returns are proportional to consumption growth volatility. Another way to see this correlation is from the fact that consumption growth is negatively skewed with the skewness of quarterly consumption growth being -0.53 and that of annual consumption growth being -1.39.[29] In this context, Yang (2009) has recently considered the importance of left skewness in long run risk models but only for durables consumption. Another study of note with regard to this relationship is that of Ferson and Constantinides (1991) who also documented a negative relationship between asset returns and past consumption growth but did so for the purpose of establishing the importance of habit formation in asset pricing. In the current paper, I extend the study of this relationship by applying it to predicting future excess market returns, particularly with the use of real time consumption data, and show that it is robust and holds out of sample.

I tabulate the results of regressing annual post 1942 excess market returns on lagged annual consumption growth, using both current and real time data, in table 36. The real time annual consumption growth is constructed in the manner described in the data section and data for it is available from 1965. The out of sample adjusted $R^2$ is calculated with the use of expanding window regressions using predicted values from 1986 so that the regression has a minimum of 20 data points for reasonably accurate estimation. The adjusted $R^2$ is calculated by comparing the sum of squared residuals obtained by regressing the realized value minus the out of sample predicted value on a constant (denoted by $SSR_{pred}$) and by regressing the realized value alone on a constant (denoted by $SSR_{null}$). This is equivalent to comparing the predictive value of the variable with a constant. The out of sample (OOS) adjusted $R^2$ is then defined as

$$OOS\ \overline{R^2} = 1 - \frac{SSR_{pred}}{SSR_{null}} \frac{df_{null}}{df_{pred}} \tag{24}$$

where $df_{null}$ is one less than the number of out of sample predictions and $df_{pred} = df_{null} - 1$ (this accounts for the degree of freedom taken up by the predictor). This is similar to the approach followed by Welch and Goyal (2008). From the tabulated results, I find that annual excess market returns are negatively related to lagged annual consumption growth and that the relation is stronger when real time data is used. Further, the relation holds out of sample, albeit

[29] However, it must be noted that the predictability here is relatively short term while the long run risk model implies longer term predictability (since consumption growth volatility is very persistent.

**Results of regressing annual excess market returns on lagged annual consumption growth**

| | Intercept | $\Delta c_t$ | $\Delta c_t^{RT}$ | Adj. $R^2$ | OOS Adj. $R^2$ |
|---|---|---|---|---|---|
| $R_{m,t+1} - R_{f,t}$ | 0.154*** (0.040) | −3.59* (1.67) | | 5.3% | |
| | 0.163*** (0.049) | | −6.54* (2.35) | 13.9% | 2.88% |
| 1/31 to 1/31 | 0.144*** (0.036) | −3.47* (1.49) | | 6.4% | |
| $R_{m,t+1} - R_{f,t}$ | 0.140*** (0.044) | | −5.62* (2.09) | 12.9% | 4.11% |

Table 36: Results of regressing (non continuously compounded) annual excess market returns on lagged annual consumption growth using both current and real time data (denoted by the superscript $RT$). Standard errors are reported in brackets. The construction of real time annual consumption growth is described in the data section of the paper and data for it is available from 1965. The out of sample adjusted $R^2$ is calculated with the use of expanding window regressions with the null being the use of a constant as in (Welch and Goyal 2008). Only predicted values from 1986 are used to make this calculation so as to provide at least 20 points to estimate the regression with reasonable accuracy. The procedure is repeated for returns from Jan 31st of the current year to Jan 31st of the next since the consumption data is released at the end of January. The population data used to convert the data to per capita values is not real time and using the fully real time aggregate consumption growth gives an OOS adjusted $R^2$ of 2.56% for end year to end year returns and 3.44% for Jan 31st to Jan 31st returns.

with a much smaller adjusted $R^2$, a result which is remarkable it is notoriously difficult to obtain a positive adjusted $R^2$ for out of sample predictions of excess market returns (Welch and Goyal 2008) (Avramov 2002).

I tabulate the corresponding results for quarterly excess market returns in table 37. Real time quarterly consumption growth is constructed as described in the data section. The negative relation found in the annual data only holds for consumption growth four quarters ago. This is probably due to consumers making consumption decisions on an annual basis (as, for example, strongly suggested by Jagannathan and Wang (2007)).[30] I find that the relation continues to hold out of sample when I use expanding window regressions. Since the magnitude of predictability is small at the quarterly level, I only use the predictions from the beginning of 1990 to calculate the out of sample adjusted $R^2$. This ensures that the regression always has a reasonably large number of points so that the coefficients can be accurately estimated.

Hence, I conclude that there is significant evidence that excess market returns are negatively related to lagged consumption growth after the structural break in 1942. The relations also hold out of sample, unlike many other predictors considered in the literature.

[30] Another alternative possibility, whose investigation I leave for future research, could be the effects of seasonal adjustment of consumption data. However, this seems unlikely as the relation holds out of sample.

**Results of regressing quarterly excess market returns on quarterly per capita consumption growth four quarters ago**

| | Intercept | $\Delta c_{t-3}$ | $\Delta c^{RT}_{t-3}$ | Adj. $R^2$ | OOS Adj. $R^2$ |
|---|---|---|---|---|---|
| $R_{m,t+1} - R_{f,t}$ | 0.029*** (0.007) | −2.39* (0.99) | | 1.9% | |
| | 0.029** (0.009) | | −4.03* (1.37) | 4.3% | 2.02% |

Table 37: Results of regressing (non continuously compounded) quarterly excess market returns on quarterly consumption growth four quarters ago using both current and real time data (denoted using the superscript $RT$). Standard errors are reported in brackets. The construction of real time quarterly consumption growth is described in the data section of the paper and data for it is available from the second quarter of 1965. The out of sample adjusted $R^2$ is calculated with the use of expanding window regressions with the null being the use of a constant as in (Welch and Goyal 2008). Only predicted values from 1990 onwards are used to make this calculation so as to provide at least 25 years of data to estimate the regression with reasonable accuracy. The population data for converting the consumption growth to a per capita value is not real time and the OOS adjusted $R^2$ using the fully real time aggregate consumption growth data is 1.58%.

# 9 Conclusion

In this paper, I show that a general long run risk model which incorporates the models of Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) implies that log price dividend ratios of assets have a strict factor structure (up to a loglinear approximation) and that these factors must be related to consumption growth and consumption growth volatility. I find that the data shows that such models could not have had a constant set of parameters over the period 1930-2008 and that there is strong evidence of a structural break in their values at about 1942. Restricting myself to post-1942 data and using the 25 Fama-French portfolios as the assets, I find that their log price dividend ratios have two significant factors, one of which is related to consumption growth volatility and the other of which is related to future dividend and real time consumption growth (as well as consumption growth after a gap of a few years).

The long run risk model considered in this paper also implies that the lagged factors and their innovations should determine asset returns. I find that cross sectional regressions using these factors do a good job of pricing not only the 25 portfolios from which they were formed but also three sets of ten portfolios based on long term reversal, short term reversal and the earnings to price ratio. I find that the coefficients obtained from the cross sectional regressions are significant and of the sign expected from the relation of the factors to consumption growth and consumption growth volatility and that the zero beta rate is not significantly different from zero. Overall, my findings indicate that long run risk models of the type considered here have potential in explaining the cross section of asset returns and that consumption growth as well as it's volatility are important.

I also find that the implication of the model (when combined with the observed negative correlation between long term consumption growth and consumption growth volatility) that consumption growth predicts future excess market returns is confirmed in the data and also that it is robust and holds both in and out of sample.

My findings also show that long run risk models might explain the relative success of factor models in the literature since the innovations to linear combinations of log price dividend ratios are similar to the returns of portfolios which are usually taken to be the factors. The crucial difference is that the long run risk models concentrate on the component of returns unexplained by predictable dividend growth rather than on returns themselves.

Recent literature (Beeler and Campbell 2009) has shown that long run risk models imply, counter-factually, significant predictability of long term consumption growth and market volatility by the market price-dividend ratio. They also imply several counter-factual properties for real risk free bond yields (Beeler and Campbell 2009). Adding more factors as in (Zhou and Zhu 2009) or (Marakani 2009b) help in resolving the predictability issues but do so only when one price-dividend ratio is used as the predictor. However, as I have shown in this paper, concentrating on relatively short term consumption growth predictability might not be sufficient as there is evidence of nonlinear relationships. This suggests that the predictability of consumption growth after a few years is of interest and I do find evidence of such predictability. The issue with real risk free and consol bond yields can also be resolved at the cost of requiring very high risk aversion values by moving to a trend stationary type model (Marakani 2009b). Long run risk models with both permanent and transitory shocks to consumption growth might provide a happy medium and resolve the issue without having to resort to high risk aversion values to explain the equity premium and future research should resolve whether this is indeed possible.

The fact that the innovations of the volatility and consumption/dividend growth factors are positively correlated to each other indicates that another promising area for future research is the theoretical analysis of long run risk models in which the consumption and dividend growth processes are negatively related to the consumption volatility process. Since expected excess returns are determined by the consumption growth volatility in long run risk models, such a model will endogenously generate higher expected excess returns in times of low consumption and dividend growth, i.e. recessions. Such a model will also ensure that cash flow risk and discount rate risk are not independent of each other as is the case in the data (Ball, Sadka, and Sadka 2009). Incorporating growth options into the dividend process of such a model will also potentially solve the equity duration puzzle pointed out by Croce, Lettau, and Ludvigson (2007). This follows from the synthesis of the postulate that growth stocks have a much larger growth option component than value stocks and the fact that the volatility factor is much more persistent than the consumption/dividend growth factor. Since growth options have a positive vega, this model will also potentially explain the result that value stocks are more exposed to long run consumption volatility shocks than growth stocks (Boguth and Kuehn 2008), a result which is otherwise inconsistent with traditional long run risk models as growth stocks are known to have a higher exposure to discount rate shocks than value stocks (Cochrane 2005) (this exposure is probably driven by short run volatility components as in (Zhou and Zhu 2009) as they are not well hedged by the growth option). The value premium can also be potentially explained by the volatility insurance provided by growth options in such a model. Such an explanation will resolve the standard puzzle that longer duration stocks have lower risk without having to adopt drastic measures such as setting the price of discount rate shocks to zero, as, for e.g., by Lettau and Wachter (2007). I am currently investigating a model incorporating growth options and correlated consumption growth and volatility processes in order to determine if it can indeed solve these puzzles in the literature.

# A Log price dividend ratios in the general long run risk model

I note that the methodology here closely follows that in (Bansal and Yaron 2004) and (Bansal, Yaron, and Kiku 2007) as there are only two common cases where solutions to models with Epstein-Zin preferences are possible. The first case, which we are interested in here, is when the returns are loglinear in the state variables and the second is when $\psi = 1$. Let $c$, $X_i, 1 \leq i \leq n$ and $V_j, 1 \leq j \leq m$ be the log consumption process, $n$ processes that determine it's conditional growth rate and $m$ processes that determine it's conditional growth rate volatility respectively. Let $d_l, l \leq 1 \leq L$ be the log dividend processes of $L$ assets (in general, the lower case variables correspond to the logarithm of the upper case variables). I assume that these quantities follow the processes

$$
\begin{aligned}
c_{t+\Delta t} =& c_t + \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t + \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}}\,(W_{t+\Delta t} - W_t) \\
&- \sum_{k=1}^{m} \varphi_{w,k}\sqrt{V_{k,t}}\,(Z_{k,t+\Delta t} - Z_{k,t})
\end{aligned} \tag{25}
$$

$$
X_{i,t+\Delta t} = X_{i,t}(1-\alpha_i \Delta t) + \varphi_{i,x}\sqrt{\sum_{j=1}^{m} \delta_{x,i,j}^2 V_{j,t}}\,(Y_{i,t+\Delta t} - Y_{i,t}), 1 \leq i \leq n \tag{26}
$$

$$
V_{i,t+\Delta t} = V_{i,t} - \kappa_i (V_{i,t} - \bar{V}_i)\Delta t + \sigma_i \sqrt{V_{i,t}}\,(Z_{i,t+\Delta t} - Z_{i,t}), 1 \leq i \leq m \tag{27}
$$

$$
\begin{aligned}
d_{l,t+\Delta t} =& d_{l,t} + \left(\mu_l + \sum_{i=1}^{n} \phi_{l,i} X_{i,t}\right)\Delta t + \pi_{l,d}\left(\Delta c_{t+\Delta t} - \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t\right) \\
&+ \sum_{i=1}^{n} \pi_{i,l,x}(X_{i,t+\Delta t} - X_{i,t}(1-\alpha_i \Delta t)) \\
&+ \sum_{j=1}^{m} \pi_{j,l,w}\sigma_j \sqrt{V_{j,t}}\,(Z_{j,t+\Delta t} - Z_{j,t}) \\
&+ \sqrt{\sum_{k=1}^{m} \delta_{l,d,k}^2 V_{k,t}}\,(B_{t+\Delta t} - B_t)
\end{aligned} \tag{28}
$$

where $W$, $Y_i, 1 \leq i \leq n$, $Z_j, 1 \leq j \leq m$ and $B$ are independent Brownian processes and $\sum_{i=1}^{m} \delta_{c,i}^2 = \sum_{j=1}^{m} \delta_{x,i,j}^2 = \sum_{k=1}^{m} \delta_{l,d,k}^2 = 1$. I have written the equations in this form (with the time step being $\Delta t$ rather than 1) to make the time scale dependence of the parameters explicit so that the connection with the continuous time solution can be made in a straightforward manner. The consumption and dividend variables are defined as rates since they are flow variables. Hence, for example, consumption from time $t$ to $t + \Delta t$ is $C_{t+\Delta t}\Delta t$.

Since the consumer preferences are of the Epstein-Zin type

$$U_t = ((1-\delta)(C_t\Delta t)^{\frac{1-\gamma}{\theta}} + \delta E_t[U_{t+\Delta t}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \tag{29}$$

where

$$\theta = \frac{1-\gamma}{1-1/\psi} \tag{30}$$

the log stochastic discount factor in discrete time can be written as

$$m_{t+\Delta t} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+\Delta t} + (\theta - 1) r_{c,t+\Delta t} \tag{31}$$

where $r_{c,t+\Delta t}$ is the continuously compounded rate of return on the wealth $W$ (which is the asset that delivers a dividend of per capita consumption at every time period) from $t$ to $t+\Delta t$. Since I assume complete markets,

$$E_t[\exp(m_{t+\Delta t} + r_{c,t+\Delta t})] = 1 \tag{32}$$

must hold.

The loglinear approximation pioneered in (Campbell and Shiller 1988) allows us to write

$$r_{c,t+\Delta t} = \nu_0 + \nu_1(w_{t+\Delta t} - c_{t+\Delta t}) - (w_t - c_t) + \Delta c_{t+\Delta t} \tag{33}$$

where

$$\nu_0 = \log(\Delta t + \exp(\overline{w-c})) - \nu_1(\overline{w-c}) \approx \exp(\overline{c-w})(1 + (\overline{c-w}))\Delta t \tag{34}$$

$$\nu_1 = \frac{1}{1+\exp(\overline{c-w})\Delta t} \approx 1 - \exp(\overline{c-w})\Delta t \tag{35}$$

(the approximation holds when $\Delta t$ is small and is only used to derive the continuous time limit) where the bar stands for the mean value. I further assume that the log wealth to consumption ratio can be written as

$$w_t - c_t = A_0 + \sum_{i=1}^{n} A_{1,i} X_{i,t} + \sum_{j=1}^{m} A_{2,j} V_{j,t} \tag{36}$$

and justify this below. (This approach is standard and followed in (Bansal and Yaron 2004), (Bansal, Yaron, and Kiku 2007) and (Zhou and Zhu 2009) as the only non-trivial models with Epstein-Zin preferences which can be solved are those where the consumption to wealth ratio is loglinear in the state variables as above or where $\psi = 1$, as in (Hansen, Heaton, and Li 2008)).

Substituting (31), (33) and (36) into (32), using the fact that

$$\log E_t[\exp A(W_{t+\Delta t} - W_t)] = \frac{A^2 \Delta t}{2} \tag{37}$$

for any $A \in \mathbb{R}$ and Wiener process $W$, and that (32) should hold for any possible attainable combination of state variables $(X_i, V_j)$, I obtain a set of equations which enable me to solve for $A_0$, $A_{1,i}, 1 \leq i \leq n$ and $A_{2,j}, 1 \leq j \leq m$. The fact that such a set of equations with non-vacuous solutions exist justifies the assumption (36).

The set of equations for $A_{1,i}$ are

$$(1-\gamma)\Delta t + \theta A_{1,i}(\nu_1(1-\alpha_i \Delta t) - 1) = 0 \tag{38}$$

so that

$$A_{1,i} = \frac{(1-\frac{1}{\psi})\Delta t}{1 - \nu_1(1-\alpha_i \Delta t)} \tag{39}$$

which, in the limit $\Delta t \to 0$, becomes $A_{1,i} = \frac{1-1/\psi}{\exp(\overline{c-w})+\alpha_i}$. This is the same result as that obtained in (Zhou and Zhu 2009), where there is only one $X$ variable, once we relate his notation of $g_1$ for $\exp(\overline{c-w})$ and allow for the negative sign which arises since he defines $A_1$ in terms of the consumption to wealth ratio. Once I set $\Delta t = 1$, $\nu_1$ to $\kappa_1$ and $\alpha_i = 1 - \rho$ (again, there being only one $X$ state variable) to match the notation in (Bansal and Yaron 2004), I find that this result also matches the one there.

The corresponding set of equations which enables us to solve for $A_{2,j}, 1 \leq j \leq m$ are

$$\begin{aligned} &\frac{(1-\gamma)^2 \delta_{c,j}^2 \Delta t}{2} + \theta A_{2,j}(\nu_1(1-\kappa_j \Delta t) - 1) \\ &+ \frac{\Delta t}{2}\left(\left(\theta \nu_1 \sum_{i=1}^{n} A_{1,i}\varphi_{x,i}\delta_{x,i,j}\right)^2 + (\theta\nu_1 A_{2,j}\sigma_j - (1-\gamma)\varphi_{w,j})^2\right) = 0 \end{aligned} \tag{40}$$

Since these equations are quadratic, there are two solutions for each $A_{2,j}$. However, one of them diverges when $\sigma_j \to 0$. Hence, the other solution is the one which is relevant to the model. It can be shown that if $\psi > 1$, $A_{2,j} < 0\,\forall j, 1 \leq j \leq m$ provided that the solutions are real.[31] The final equation, which allows us to solve for $A_0$, is

$$\theta\left(\log\delta + \nu_0 + (\nu_1 - 1)A_0 + \nu_1 \sum_{j=1}^{m} A_{2,j}\kappa_j \Delta t \bar{V}_j\right) + (1-\gamma)\mu\Delta t = 0 \tag{41}$$

Putting the values for $A_0$, $A_{1,i}, 1 \leq i \leq n$ and $A_{2,j}, 1 \leq j \leq m$ into (36) and

[31] If the solutions are complex, then this methodology is unable to solve for the wealth to consumption ratio and the stochastic discount factor for that set of parameters. This can be rectified at the cost of admitting negative volatilities even in continuous time by changing the volatility process to an Ornstein-Uhlenbeck one as in (Bansal and Yaron 2004).

using (33) and (31), we obtain the log stochastic discount factor

$$\begin{aligned}
m_{t+\Delta t} =& \Delta t \left( \Gamma_0 + \sum_{i=1}^{n} \Gamma_{1,i} X_{i,t} + \sum_{j=1}^{m} \Gamma_{2,j} V_{j,t} \right) \\
&- \lambda_c \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}} (W_{t+\Delta t} - W_t) \\
&- \sum_{i=1}^{n} \lambda_{x,i} \sqrt{\sum_{j=1}^{m} \delta_{x,j}^2 V_{j,t}} (Y_{i,t+\Delta t} - Y_{i,t}) \\
&- \sum_{j=1}^{m} \lambda_{v,j} \sqrt{V_{j,t}} (Z_{j,t+\Delta t} - Z_{j,t})
\end{aligned} \tag{42}$$

where $\Gamma_{1,i} = 1/\psi$, $\lambda_c = \gamma$ and $\lambda_{x,i} = \frac{\gamma - 1/\psi}{1 - \nu_1(1 - \alpha_i \Delta t)}$. The expression for $\lambda_{v,j}$ is complicated and does not directly concern us here. Hence, I do not present it but note that, as expected, $\lambda_{v,j} < 0$ if $\gamma - 1/\psi > 0$.

Using the process for dividend growth (28), one can use a similar loglinear approximation to write the return for asset $l$

$$r_{l,t+\Delta t} = \nu_{0,l} + \nu_{1,l}(p_{l,t+\Delta t} - d_{l,t+\Delta t}) - (p_{l,t} - d_{l,t}) + \Delta d_{l,t+\Delta t} \tag{43}$$

where

$$\begin{aligned}
\nu_{0,l} &= \log(\Delta t + \exp(\overline{d_l - p_l})) - \nu_{1,l}(\overline{p_l - d_l}) \\
&\approx \exp(\overline{d_l - p_l})(1 + \overline{d_l - p_l})\Delta t
\end{aligned} \tag{44}$$

$$\nu_{1,l} = \frac{1}{1 + \exp(\overline{d_l - p_l})\Delta t} \approx 1 - \exp(\overline{d_l - p_l})\Delta t \tag{45}$$

As before, I assume that $\log\left(\frac{P_t}{D_t}\right)$ can be written as

$$\log\left(\frac{P_{l,t}}{D_{l,t}}\right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i} X_{i,t} + \sum_{j=1}^{m} A_{2,l,j} V_{j,t} \tag{46}$$

I put (46) into (43) and use the fact that (32) must hold for any possible attainable combination of state variables $(X_i, V_j)$ to obtain a set of equations which enable me to solve for $A_{0,l}$, $A_{1,l,i}, 1 \le i \le n$ and $A_{2,l,j}, 1 \le j \le m$. The fact that such a set of equations with non-vacuous solutions exist justifies the assumption (46).

The equations for $A_{1,l,i}, 1 \le i \le n, 1 \le l \le L$ are

$$(\phi_{l,i} - 1/\psi)\Delta t - A_{1,l,i}(1 - \nu_{1,l}(1 - \alpha_i \Delta t)) = 0 \tag{47}$$

which give

$$A_{1,l,i} = \frac{(\phi_{l,i} - 1/\psi)\Delta t}{1 - \nu_{1,l}(1 - \alpha_i \Delta t)} \tag{48}$$

As with the solution for $A_{1,i}, 1 \leq i \leq n$, this solution agrees with the continuous time one with $n = 1, m = 2$ in (Zhou and Zhu 2009) and the discrete time one with $n = m = 1$ in (Bansal and Yaron 2004) and (Bansal, Yaron, and Kiku 2007).

The equations for $A_{2,l,j}, 1 \leq j \leq m, 1 \leq l \leq L$ are quadratic in nature and fairly complex (as for $A_{2,j}$, the solutions which do not diverge as $\sigma_j \to 0$ are chosen). Since their precise structure does not concern us here, I do not include them for brevity. Similarly, I do not include the equation for $A_{0,l}, 1 \leq l \leq L$.[32]

It must be noted that, as the equations for $A_{2,j}, 1 \leq j \leq m$ and $A_{2,l,j}, 1 \leq j \leq m, 1 \leq l \leq L$ are quadratic in nature, real solutions are not guaranteed. My numerical experiments indicate that this is not a serious concern as several sets of reasonable parameter values do not give rise to this problem (this is also shown in (Zhou and Zhu 2009)). If this is a concern, we can replace the volatility processes by Ornstein-Uhlenbeck ones as in (Bansal and Yaron 2004) and (Bansal, Yaron, and Kiku 2007). However, these volatility processes suffer from the problem of admitting negative values even in continuous time. This can be quite serious, even for some common parameter values, as pointed out in (Beeler and Campbell 2009). The square root processes used here can also give rise to negative values in discrete time but the probability of this occurring for reasonable parameter values is minuscule and my numerical experiments confirm this. Since both ways of modeling volatility have issues but have received wide attention in the literature and there is no known alternative for which analytical solutions can be derived, I use results which apply when either of them is used.

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[32] They are available upon request from the author.

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