Long Run Risks & Price/Dividend Ratio Factors*

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ABSTRACT

We show that long run consumption risk models imply that the covariance matrix of the logarithm of price to dividend (P/D) ratios of stocks has a strict factor structure. Factor analysis of the P/D ratios of 25 portfolios formed by sorting stocks based on their size and book to market ratio during the 1943 to 2008 reveals two significant factors. Consistent with theory, these factors predict growth in US aggregate consumption & dividends and consumption growth volatility, and explain the cross section of average excess returns on portfolios based on size, book/market, long term reversal, short term reversal, and earnings to price ratios.

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Recent research in asset pricing has focused on models of dynamic economies in order to provide a better understanding of the underlying economic forces behind various empirical regularities observed in financial markets. Table (I) summarizes three of the empirical regularities that have received attention: the high historical average risk premium on corporate equities, large cross sectional variation in historical average returns across various asset classes, and low risk free rate.

Table I
Summary of historical returns on various stock portfolios

Descriptive statistics of the real continuously compounded returns and real dividend growth rates of ten assets over the period 1950-2008. The all stock index is the value weighted index of all stocks on the NYSE, NASDAQ and AMEX, and the data are from CRSP. Definitions of growth, value, small cap, large cap, small growth, small value, large growth and large value stocks are provided in Section IV. Standard deviations are computed using annual data, and nominal returns are deflated by the inflation rate as measured by the CPI to get the corresponding real returns. β_{ASI} is the beta of the asset with respect to the all stock index.

	Returns		Div. growth rate		β_{ASI}
	Mean (%)	Std. Dev. (%)	Mean (%)	Std. Dev. (%)	
Risk free rate	1.20	2.3			
All Stock Index (US)	6.11	18.0	1.51	5.9	1.00
Growth stocks (B1)	4.65	21.0	0.49	15.5	1.09
Value stocks (B10)	9.49	24.0	4.77	21.3	1.03
Small stocks (S1)	7.42	27.5	4.16	15.1	1.08
Large stocks (S10)	5.71	19.5	0.96	6.6	0.94
Small growth (FF1)	4.10	25.9	-0.98	19.3	1.31
Small value (FF3)	11.17	22.4	6.51	15.6	1.02
Large growth (FF4)	5.61	18.5	1.05	10.1	1.03
Large value (FF6)	8.60	20.3	3.04	11.3	0.93

In general, an asset with lower systematic risk, i.e., an asset with a larger fraction of its payoff occurring during bad economic times, will be more valuable and thus earn a lower return on average. An asset pricing model takes a stand on how to classify time periods into good and bad ones. In a static one period economy good times will correspond to periods of higher consumption and higher aggregate wealth. Lucas (1978) and Breeden (1979) showed that good times will also correspond to high aggregate consumption growth in dynamic multi-period economies when there is a representative investor with a standard separable utility function. Rubinstein (1976) showed that consumption growth can be replaced by the aggregate wealth return when the representative investor has a logarithmic utility function and the investment opportunity set is time-varying; and when the representative investor has a constant relative risk aversion (CRRA) utility function and the investment opportunity set is constant over time. Merton (1973) developed the intertemporal capital asset pricing model (ICAPM) which holds when the investment opportunity set (i.e., expected returns, variances/covariances, and higher moments of available assets) varies stochastically over time and the representative investor has standard time separable utility function. In such an economy, Merton (1973) showed that good economic times will depend not only on the return on the aggregate wealth portfolio but also on state variables that characterize future investment opportunities.

Whereas an investor with standard time separable utility function will be indifferent to the temporal resolution of uncertainty, stylized facts suggest that most investors prefer earlier resolution of uncertainty. Kreps and Porteus (1978) developed an interesting alternative to the standard time separable utility function that allows for such a preference over the temporal resolution of uncertainty. Epstein and Zin (1989) and Weil (1990) derived expressions that help classify the states of the economy based on the value of security payoffs in those states for a specific parametric class of the Kreps and Porteus (1978) utility functions. In particular, they showed that for classifying economic times as good or bad, it is necessary to know the growth rate in aggregate consumption as well as the return on the aggregate wealth portfolio. Skiadas (2009) showed that these variables are sufficient even when investor preferences belong to the more general scale invariant Kreps-Porteus class.

Campbell (1993) showed that Merton (1973)'s ICAPM continues to approximately hold even when the utility function of the marginal investor belongs to the Epstein and Zin (1989) class provided the economy is homoskedastic. In particular, he showed that the return on the aggregate wealth portfolio together with state variables that help forecast future returns on the aggregate wealth portfolio are sufficient to classify the economy into relatively good and bad times. Bergman (1985) derived related results when the representative investor's utility function is inter-temporally dependent through habit.

Roll (1977) pointed out the difficulties associated with measuring the return on the aggregate wealth portfolio. Campbell (1996) and Jagannathan and Wang (1996) further argued that it is important to account for the return on human capital when measuring the return on the aggregate wealth portfolio. The linear factor pricing model of Ross (1976) avoids the need to measure the return on the aggregate wealth portfolio but can only price assets whose returns have a linear factor structure. Bansal and Yaron (2004) addressed this issue by showing that, under suitable assumptions it is possible to replace current and future returns on the aggregate wealth portfolio by aggregate consumption growth and changes in its future means and variances, by using the methodology of Campbell (1993).

Bansal and Yaron (2004) further showed that their model for classifying economic times into relatively good and bad ones is consistent with a wide variety of asset market facts. While the Bansal and Yaron (2004) model has received wide attention in the literature, it has a major shortcoming. Since even small shocks to consumption can be highly economically significant provided they are sufficiently persistent, precise measurement of critical model parameters is necessary for the empirical validation of the model, and this is not feasible given the limited length of time for which consumption data is available.[1] Bansal and Yaron (2004) therefore rely on suitably calibrated parameter values to make

[1] Croce, Lettau, and Ludvigson (2007) show that precise measurements may be difficult even with an infinite amount of consumption data.

their arguments. The downside, as pointed out by Constantinides and Ghosh (2008) and others, is that the validity of the arguments in Bansal and Yaron (2004) depend on parameter values that cannot be estimated precisely enough to change the views of those with reasonable and sufficiently strong priors. Because of that their findings have been the subject of much debate.

In this paper we use a method for evaluating the empirical support for the Bansal and Yaron (2004) model that overcomes this shortcoming. In particular we show that when a general version of the Bansal and Yaron (2004) model holds, the factors that determine the stochastic process for aggregate consumption can be estimated by factor analysis of the log P/D ratios of a collection of stocks. The advantage of this approach is that (a) we do not have to measure consumption or market wealth and in addition, (b) not all investors need to be at the margin. This is in contrast with the methods used by Bansal, Yaron, and Kiku (2007), Constantinides and Ghosh (2008), Ferson, Nallareddy, and Xie (2009) and others who make use of the stock market log P/D ratio and the real risk free rate for this purpose. The use of several log P/D ratios instead of these quantities possesses two significant advantages. The first advantage is that we do not need a long time series of observations of the real risk free rate. We show, in an appendix, that this is a significant advantage because the probability of rejecting the Bansal and Yaron (2004) model is high, even when it is the correct one, when realistic measurement errors in the real risk free rate and consumption growth are taken into account. The second advantage is that this methodology allows for more underlying factors than those considered by Bansal and Yaron (2004). This is because we do not require the number of factors to be specified *a priori*, and determine that number from data. Our use of factors is thus in the spirit of the stand taken by Roll and Ross (1980).

Using our approach we find empirical support for the Bansal and Yaron (2004) view that modeling long run risks in the presence of preference for temporal resolution of uncertainty may be necessary to explain the cross section of asset returns in the following sense: the factors that help explain the cross section of returns on a variety of stock portfolios are also helpful in forecasting changes in future consumption and dividend growth as well as consumption growth volatility.[2,3] These conclusions are robust to the recent critique of factor models by Kleibergen (2010).

Our results also provide another possible explanation for the results of Jagannathan and Wang (2007) and Jagannathan, Marakani, Takehara, and Wang (2011) who find that growth in consumption from the final quarter of a tax year (Q4 in the US) to the next (i.e. Q4 of the next year) explains the cross section of stock returns across the same period while the growth in annual consumption does not. Jagannathan and Wang (2007) and Jagannathan, Marakani, Takehara, and Wang (2011) argue that this is probably due to the existence of a large section of investors who only trade at the end of the tax year. We however find, in addition, that the Q4-Q4 consumption growth that they use may also be serving as a proxy for the log P/D factor innovations since the correlation of consumption growth with these factor innovations is much higher when consumption growth is measured from the last quarter of one tax year to the next. We support this argument by showing that the log P/D factor innovations drive out consumption growth in the cross sectional regression.

The rest of the paper is organized as follows. Section I summarizes the related literature. Section II introduces our version of the long run risk model which encompasses the ones of Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009). Section III discusses the implication of the model for the factor structure of log P/D ratios and asset pricing. Section IV describes the data. Section V develops the econometric specifications and discusses the empirical findings. Section VI concludes.

[2] Note that while temporal resolution of uncertainty may matter for some asset classes, it may not matter for others.

[3] Note that, strictly speaking, the factors jointly predict consumption and dividend growth but only *track* contemporaneous consumption growth volatility. However, for ease of exposition, we loosely refer to tracking consumption growth volatility as predicting it. The reader will note that the empirical analysis of the relation between the factors and consumption growth in this paper uses the tracking rather than predictive property.

I. Related Literature

The literature on consumption and factor based asset pricing models is vast and a review of them is beyond the scope of this paper. We will therefore limit our discussion to papers that are immediately relevant to our work.

In closely related work, Bansal, Yaron, and Kiku (2007), Constantinides and Ghosh (2008) and Ferson, Nallareddy, and Xie (2009) assume that the number of factors is fixed and equal to two and that the real risk-free rate and the P/D ratio of the aggregate stock market portfolio are observed without error. In contrast, we allow for measurement errors in P/D ratios that may arise due to temporary price fluctuations around the fundamental value due to features of the economy that we do not model; and we do not require that the real risk free rate to be observable. Since we rely on weaker restrictions imposed by the model and do not require knowledge of the parameters of the consumption and dividend processes as in Constantinides and Ghosh (2008), our findings, while consistent with the Bansal and Yaron (2004) long run risk model, cannot rule out other competing models.[4]

We believe that this is not a major shortcoming, since, as Constantinides and Ghosh (2008) point out, the parameters cannot be precisely estimated with the limited amount of available data. While constraining the feasible set of values for some of the parameters can reduce the dimensionality of the problem and thus partially alleviate the issue, it must be noted that these constrained values are generally arbitrary and not data dependent. If these values are incorrectly chosen, spurious rejection of the underlying model can occur, particularly since the stochastic discount factor in long run risk models exhibits sensitive dependence on them. In this context, it should be noted that the chosen constrained values of the parameters differ across studies. For example, the stochastic discount factor for the Bansal and Yaron (2004) model derived by Constantinides and

[4]Although Bansal, Yaron, and Kiku (2007) take into account the dependence of the market prices of risk on model parameters, they do not impose those restrictions when estimating the pricing relation.

Ghosh (2008) does not extend to the parametrization used by Bansal, Yaron, and Kiku (2007). This is because Bansal and Yaron (2004) and Constantinides and Ghosh (2008) set the parameter governing the correlation between the innovations in the consumption and dividend growths to zero while Bansal, Yaron, and Kiku (2007) do not. Ferson, Nallareddy, and Xie (2009) also take a position similar to ours in this regard as their approach also does not require estimating the parameters describing the consumption and dividend processes. However, they make a different set of assumptions which we do not; they assume that the real risk free rate is observed without error.

An alternative long run risk formulation which we do not explore in this paper is due to Hansen, Heaton, and Li (2008).[5] Hansen, Heaton, and Li (2008) show that the value premium, but not the equity premium, can be explained using this formulation and that a relatively high risk aversion value of around 30 is required for this purpose.[6] Malloy, Moskowitz, and Vissing-Jørgensen (2009) show that a closely related formulation is capable of explaining both the equity and value premia when stockholder consumption is used and that a relatively low relative risk aversion value of about 15 is sufficient for this purpose. In addition, they show that the study of Parker and Julliard (2005), whose results were originally explained on the basis of consumption adjustment costs and measurement error in consumption data, can also be cast into this framework. Despite the impressive results obtained using this approach, we do not explore it further because it does not easily accommodate stochastic volatility which, as shown by the studies of Bansal and Yaron (2004), Boguth and Kuehn (2008), Zhou and Zhu (2009) and Beeler and Campbell (2009), plays a crucial role in pricing assets when the temporal resolution of uncertainty matters.

[5]In Hansen, Heaton, and Li (2008), unlike Bansal and Yaron (2004), shocks to consumption growth need not be persistent. This is a plus since Malloy, Moskowitz, and Vissing-Jørgensen (2009) find that consumption growth of stock holders, is much less persistent than aggregate consumption growth.

[6]If aggregate rather than per capita consumption is used, as in Hansen, Heaton, and Li (2008), the risk aversion required is only about 20. It is, however, standard to use per capita consumption in the literature as pointed out by Marakani (2009).

An alternative method of studying Epstein-Zin preferences has been pioneered by Chen, Favilukis, and Ludvigson (2007) who do not impose any restrictions such as the long run risk model in their estimations. They are able to do this by approximating the continuation utility, which otherwise requires additional assumptions to estimate, with the use of splines. They find that Epstein-Zin preferences, with the use of stockholder consumption, is able to explain the cross-section of stock returns with a modest risk aversion of 17. While interesting, we think that the imposition of the long run risk model adds value as intuition suggests that agents are worried about the long term future and that taking this prior into account is important.

Our study is also related to the consumption-cashflow based studies such as the ones by Bansal, Dittmar, and Lundblad (2005), Bansal, Dittmar, and Kiku (2009), Lettau and Wachter (2007), Da (2009) and others but goes beyond them in adding stochastic volatility.

Yang (2011) is another related study which analyzes the long run risk of durable consumption. In contrast, we analyze long run risk with the more traditional measure of consumption which only considers non-durables and services.

II. The Long Run Risk Model

We consider the following long run risk model which accommodates the specifications proposed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) as special cases.[7] Let c, $X_i, 1 \leq i \leq n$ and $V_j, 1 \leq j \leq m$ be the log consumption process, n processes that determine it's conditional growth rate and m processes that determine the volatility of it's conditional growth rate respectively. Let $d_l, l \leq 1 \leq L$ be the log dividend processes of L portfolios (in general, the lower case

[7]Note that the volatility process has to be modified to an Ornstein-Uhlenbeck one to accommodate the first two specifications. This modification does not affect any of the fundamental theoretical results or empirical analysis.

variables correspond to the logarithm of the upper case variables). We assume that these quantities follow the processes

$$\begin{aligned} c_{t+\Delta t} =& c_t + \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t + \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}}\,(W_{t+\Delta t} - W_t) \\ & - \sum_{k=1}^{m} \varphi_{w,k} \sqrt{V_{k,t}}\,(Z_{k,t+\Delta t} - Z_{k,t}) \end{aligned} \tag{1}$$

$$X_{i,t+\Delta t} = X_{i,t}(1 - \alpha_i \Delta t) + \varphi_{i,x} \sqrt{\sum_{j=1}^{m} \delta_{x,i,j}^2 V_{j,t}}\,(Y_{i,t+\Delta t} - Y_{i,t}), 1 \leq i \leq n \tag{2}$$

$$V_{i,t+\Delta t} = V_{i,t} - \kappa_i (V_{i,t} - \bar{V}_i)\Delta t + \sigma_i \sqrt{V_{i,t}}\,(Z_{i,t+\Delta t} - Z_{i,t}), 1 \leq i \leq m \tag{3}$$

$$\begin{aligned} d_{l,t+\Delta t} = & d_{l,t} + \left(\mu_l + \sum_{i=1}^{n} \phi_{l,i} X_{i,t}\right)\Delta t \\ & + \pi_{l,d} \left(\Delta c_{t+\Delta t} - \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t\right) \\ & + \sum_{i=1}^{n} \pi_{i,l,x} (X_{i,t+\Delta t} - X_{i,t}(1 - \alpha_i \Delta t)) \\ & + \sum_{j=1}^{m} \pi_{j,l,w} \sigma_j \sqrt{V_{j,t}}\,(Z_{j,t+\Delta t} - Z_{j,t}) + \sqrt{\sum_{k=1}^{m} \delta_{l,d,k}^2 V_{k,t}}\,(B_{t+\Delta t} - B_t) \end{aligned} \tag{4}$$

where W, $Y_i, 1 \leq i \leq n$, $Z_j, 1 \leq j \leq m$ and B are independent Wiener processes and $\sum_{i=1}^{m} \delta_{c,i}^2 = \sum_{j=1}^{m} \delta_{x,i,j}^2 = \sum_{k=1}^{m} \delta_{l,d,k}^2 = 1$ (as pointed out by Zhou and Zhu (2009), these variables are necessary to ensure that the market volatility is decoupled from the consumption growth volatility as is the case in the data). The basic time interval of the process is assumed to be the same as that for which consumption is observed. This ensures that the stochastic discount factor can be related to the innovations in the processes c, X and V. If the basic time interval of the process is smaller than that for which consumption is observed, time aggregation effects prevent the calculation of the stochastic discount factor as shown by Bansal, Yaron, and Kiku (2007).[8] We note that at least some of the α_i and ν_i must be small for the risks to be long lived and therefore carry a high price.

In the above equations, consumption is defined as a rate rather than per period so that consumption from time t to $t+\Delta t$ is $C_{t+\Delta t}\Delta t$ and log consumption from t to $t+\Delta t$ is $c_{t+\Delta t} + \log \Delta t$. While this generally makes no difference as the log consumption is just offset by a constant, it ensures that the continuous time limit exists and (as we show in appendix A) also makes it easy to obtain it. It further shows that the solution method of Bansal and Yaron (2004) is general enough to apply to continuous time models such as the one in Zhou and Zhu (2009) and that the less general continuous time methods are not required to solve such long run risk models. One consequence of this definition is that the log-linearization constants depend on Δt. This is due to the fact that consumption and dividends are flow variables whose magnitude depends on the time interval (the shorter the time interval, the smaller the consumption and dividend). This fact implies that the log-linearization constants, which are functions of the average wealth to consumption or price to dividend ratio, are inversely related to the time scale. This explains the dependence of these log-linearization constants, and hence the market prices of risk, on the time unit chosen in the formulae below.

This long run risk process, when written in continuous time, incorporates the one proposed by Zhou and Zhu (2009) as a special case (specifically with $n = 1$, $m = 2$ and $\varphi_{w,i} = 0, 1 \leq i \leq m$). When the volatility process (3) is modified to an Ornstein-Uhlenbeck one plus a constant by modifying the second term to $\sigma_{i,w}\Delta Z_{i,t+1}$ and the final term in (1) to $-\sum_{k=1}^{m} \varphi_{w,k}(Z_{k,t+\Delta t} - Z_{k,t})$, it incorporates the ones proposed by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007) as special cases (specifically with $n = 1$, $m = 1$, $\varphi_{w,i} = 0, 1 \leq i \leq m$, $\pi_{i,l,x} = 0, 1 \leq i \leq n, 1 \leq l \leq L$ and $\pi_{i,l,w} = 0, 1 \leq i \leq m, 1 \leq l \leq L$).

[8]It must be noted that, for many models, reasonable approximations can be often made even in the presence of time aggregation.

Consumers in the model have Epstein-Zin preferences (Epstein and Zin 1989)

$$U_t = ((1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \tag{5}$$

As noted in Bansal and Yaron (2004), we need $\gamma > 1/\psi$ to generate a positive equity risk premium as expected dividend growth is positively related to expected consumption growth (as noted by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007), Bansal, Dittmar, and Lundblad (2005), Bansal, Dittmar, and Kiku (2009) and others). This implies that they prefer early resolution of uncertainty and that shocks to expected consumption growth carry a positive price of risk (as pointed out by Kaltenbrunner and Lochstoer (2010)) which is high if the expected consumption growth is persistent. This high price of risk results in a high equity premium and low risk-free rate.

III. Factor structure of log P/D ratios

In appendix A, we show, using the approach of Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007), that this long run risk model implies that

$$\log\left(\frac{P_{l,t}}{D_{l,t}}\right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i}X_{i,t} + \sum_{j=1}^{m} A_{2,l,j}V_{j,t} \tag{6}$$

where $P_{l,t}$ is the price of portfolio l, $A_{1,l,i} = \frac{(\phi_{l,i}-1/\psi)\Delta t}{1-\nu_{1,l}(1-\alpha_i\Delta t)}, 1 \le i \le n$ ($\nu_{1,l}$ being a log-linearization constant which is endogenously determined in the model) and where the expressions for $A_{2,l,j}, 1 \le j \le m$ are derived in appendix A. This generalizes the equivalent results by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) to the situation when there are multiple state variables describing predictable consumption growth and consumption growth volatility.

Since the real risk-free rate can be viewed as a special type of dividend-price ratio, it also follows that

$$r_{f,t} = A_{0,f} + \sum_{i=1}^{n} A_{1,f,i} X_{i,t} + \sum_{j=1}^{m} A_{2,f,j} V_{j,t} \tag{7}$$

where $A_{1,f,i} = 1/\psi$.

We show that the log stochastic discount factor for this model is given by

$$\begin{aligned} m_{t+\Delta t} =& \Delta t \left(\Gamma_0 + \sum_{i=1}^{n} \Gamma_{1,i} X_{i,t} + \sum_{j=1}^{m} \Gamma_{2,j} V_{j,t} \right) \\ & - \alpha_c \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}} (W_{t+\Delta t} - W_t) \\ & - \sum_{i=1}^{n} \alpha_{x,i} \sqrt{\sum_{j=1}^{m} \delta_{x,i,j}^2 V_{j,t}} (Y_{i,t+\Delta t} - Y_{i,t}) \\ & - \sum_{j=1}^{m} \alpha_{v,j} \sqrt{V_{j,t}} (Z_{j,t+\Delta t} - Z_{j,t}) \end{aligned} \tag{8}$$

where $\Gamma_{1,i} = -1/\psi$, $\alpha_c = \gamma$ and $\alpha_{x,i} = \frac{\gamma - 1/\psi}{1-\nu_1(1-\alpha_i \Delta t)}$.[9] The expression for $\alpha_{v,j}$ is complicated and does not directly concern us here, but we note that it was shown by Bansal and Yaron (2004) that $\alpha_{v,j} < 0$ if $\gamma - 1/\psi > 0$ and $\psi > 1$.

The relatively simple form of $\alpha_{x,i}$ implies that it can used together with a reasonable approximation for $1 - \nu_1 = \frac{\exp(\overline{c-w})\Delta t}{1+\exp(\overline{c-w})\Delta t} \approx \exp(\overline{c-w})\Delta t$ to estimate $\gamma - 1/\psi$ once a component X_i is identified. The estimation of ν_1, which can at best be done heuristically, is a cost that has to be paid when the parameters are not explicitly specified. It must be cautioned that this estimate is likely to be imprecise due to it's indirect nature but it is still useful in that it allows to relate the empirical results back to the underlying preferences.

[9] Note that the value of $\alpha_{x,i}$ depends on Δt but the risk premium does not. $\alpha_{x,i}$ varies inversely with Δt as $1 - \nu_1$ is proportional to Δt. Since the risk premium due to this risk is given by the product of $\alpha_{x,i}$ and the covariance between the return and the innovation to X_i which is proportional to Δt, the inverse relationship between $\alpha_{x,i}$ and Δt implies that the risk premium is independent of it.

In the case of no measurement error, (6) can be inverted to express the state variables (X_i, V_j) as a linear combinations of log P/D ratios. This enables the expression of the log stochastic discount factor as

$$m_{t+\Delta t} = \Delta t \left(\tilde{\Xi}_0 + \sum_{i=1}^{m+n} \tilde{\Xi}_{1,i} F_{i,t} \right) - \alpha_c (c_{t+\Delta t} - c_t) - \sum_{i=1}^{m+n} \alpha_{q,i} IF_{i,t+\Delta t} \tag{9}$$

where F_i and IF_i, $1 \leq i \leq n+m$ are the $n+m$ principal components of the log P/D ratios (or, equivalently, any linear combination of $n+m$ log P/D ratios) and their innovations respectively.

(6) implies that the log P/D ratios of assets follow a strict factor structure (up to the loglinear approximation) in the model.[10] Since log P/D ratios are not exact linear combinations of a small number of factors in the data, we use a slightly modified relation in our empirical work. This relation is

$$\log\left(\frac{P_{l,t}}{D_{l,t}}\right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i} X_{i,t} + \sum_{j=1}^{m} A_{2,l,j} V_{j,t} + \epsilon_{l,t} \tag{10}$$

where $\epsilon_{l,t} \sim N(0, V_e)$ are i.i.d. $\epsilon_{l,t}$ can be thought of as deviations that arise due to market imperfections such as illiquidity or due to the existence of incompletely diversified idiosyncratic factors. In section V, we show that, given the assumed error structure, principal component analysis (or singular value decomposition) can be used to estimate the linear subspace that spans the $n+m$ factors once $n+m$ is specified and that statistical tests suggested in the literature can be used to estimate $n+m$ from the data.

This differs from the methodology used by Bansal, Yaron, and Kiku (2007) and Ferson, Nallareddy, and Xie (2009) in estimating the linear subspace of the factors with the use of several log P/D ratios rather than the projection of the realized long term consumption growth and its volatility on the log market P/D ratio and the real risk free

[10] The reader must note that these factors are different though related to the pricing factors discussed below. This similar terminology for the two different types of quantities is standard but unfortunate.

rate. We show in the appendix B, by using Monte Carlo simulations of the long run risk model, that our methodology produces much fewer spurious rejections of the model when reasonable measurement errors in consumption growth and the real risk free rate are taken into account.

The standard asset pricing relation

$$E_t[\exp(m_{t+\Delta t} + r_{i,t+\Delta t})] = 1 \tag{11}$$

together with (9) implies a nonlinear pricing relationship involving the log P/D factors, their innovations and returns which we investigate using GMM in the empirical section of this paper. We also approximate this nonlinear relationship by a linear beta pricing one and examine it in the empirical study. We do so because such linear beta pricing models are well studied in the literature and are easier to intuitively understand (two studies that take this approach in the context of long run risk models are those of Malloy, Moskowitz, and Vissing-Jørgensen (2009) and Ferson, Nallareddy, and Xie (2009)).

Note that the model we have specified implies that $m_{t+\Delta t}$ and $r_{i,t+\Delta t}$ are conditionally normally distributed. Hence, (11) can be written as

$$\log E_t[\exp(m_{t+\Delta t} + r_{i,t+\Delta t})] = E_t[m_{t+\Delta t} + r_{i,t+\Delta t}] + \frac{1}{2}\text{Var}_t[m_{t+\Delta t} + r_{i,t+\Delta t}] = 0$$

or

$$E_t[r_{i,t+\Delta t}] + \frac{1}{2}\text{Var}_t[r_{i,t+\Delta t}] + E_t[m_{t+\Delta t}] + \frac{1}{2}\text{Var}_t[m_{t+\Delta t}] + \text{Cov}_t(m_{t+\Delta t}, r_{i,t+\Delta t}) = 0 \tag{12}$$

Since

$$r_{f,t} = -\log E_t[\exp(m_{t+\Delta t})] = -E_t[m_{t+\Delta t}] - \frac{1}{2}\text{Var}_t[m_{t+\Delta t}] \tag{13}$$

and $\text{Var}_t[r_{i,t+\Delta t}] = \text{Var}_t[r_{i,t+\Delta t} - r_{f,t}]$ (as $r_{f,t} \in \mathcal{F}_t$), (12) is equivalent to

$$E_t[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}_t[r_{i,t+\Delta t} - r_{f,t}] + \text{Cov}_t(m_{t+\Delta t}, r_{i,t+\Delta t} - r_{f,t}) = 0 \tag{14}$$

Taking the unconditional expectation of the above equation and combining it with the identity $\text{Var}[X] = E[\text{Var}_t[X]] + \text{Var}[E_t[X]]$ (with $X = r_{i,t+\Delta t} - r_{f,t}$) and the identity

$$E[\text{Cov}_t(X_{t+\Delta t}, Y_{t+\Delta t})] = \text{Cov}(X_{t+\Delta t} - E_t[X_{t+\Delta t}], Y_{t+\Delta t}) \tag{15}$$

with $X = m, Y = r_i - r_f$, we obtain the unconditional relationship[11]

$$\begin{aligned} &E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] - \frac{1}{2}\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]] + \\ &\text{Cov}[m_{t+\Delta t} - E_t[m_{t+\Delta t}], r_{i,t+\Delta t} - r_{f,t}] = 0 \end{aligned} \tag{16}$$

Substituting out $m_{t+\Delta t}$ using (9) gives us

$$\begin{aligned} E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] - \frac{1}{2}\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]] \\ = \beta_{\Delta c}\lambda_{\Delta c} + \sum_{j=1}^{n+m} \beta_{IF_j}\lambda_{IF_j} \end{aligned} \tag{17}$$

The above equation cannot be directly used as a standard linear beta pricing relationship due to the presence of the third term in it. We will therefore have to account for that term in order to obtain a usable pricing relationship. For this purpose, we use the following approach. We note that $E_t[r_{i,t+\Delta t} - r_{f,t}]$ must be a function of the X and V state variables since they completely describe the state of the economy in our model. Hence, we allow $\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]]$ to be an affine function of both X and V, or equivalently, an affine function of the log P/D factors F, in our empirical specification.

[11]Malloy, Moskowitz, and Vissing-Jørgensen (2009) work with the relationship $E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t} - E_t[r_{i,t+\Delta t} - r_{f,t}]] + \text{Cov}[m_{t+\Delta t} - E_t[m_{t+\Delta t}], r_{i,t+\Delta t} - r_{f,t}] = 0$ which follows from (14) as $\text{Var}[r_{i,t+\Delta t} - r_{f,t} - E_t[r_{i,t+\Delta t} - r_{f,t}]] = \text{Var}_t[r_{i,t+\Delta t} - r_{f,t}]$. This approach requires a stand on the set of variables that determine the expected excess returns of the various assets as well as on the functional form of the relation between the variables and these expected excess returns.

We then make use of the fact that the innovations to $r_{i,t+\Delta t} - r_{f,t}$ are orthogonal to $E_t[r_{i,t+\Delta t} - r_{f,t}]$ to write

$$\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]] = \text{Cov}(E_t[r_{i,t+\Delta t} - r_{f,t}], r_{i,t+\Delta t} - r_{f,t}) \tag{18}$$

and so express $\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]]$ in terms of the covariance between the returns and the log P/D factors F so that it can be absorbed into the linear beta pricing relation. This then leads to the usable linear beta pricing relation[12]

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \beta_{\Delta c}\lambda_{\Delta c} + \sum_{j=1}^{n+m} \beta_{F_j}\lambda_{F_j} + \sum_{j=1}^{n+m} \beta_{IF_j}\lambda_{IF_j} \tag{19}$$

An alternative approach[13] is to set $\text{Var}[E_t[r_{i,t+\Delta t} - r_{f,t}]]$ to zero. This gives the following approximate unconditional relationship

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] + \text{Cov}[m_{t+\Delta t} - E_t[m_{t+\Delta t}], r_{i,t+\Delta t} - r_{f,t}] \approx 0 \tag{20}$$

Substituting $m_{t+\Delta t}$ using (9) gives us a linear beta pricing relationship where the factors are contemporaneous consumption growth and the innovations to the identified principal components of the log P/D ratios[14]

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \beta_{\Delta c}\lambda_{\Delta c} + \sum_{j=1}^{n+m} \beta_{IF_j}\lambda_{IF_j} \tag{21}$$

where IF_i stand for the innovations to the principal components or factors of the log P/D ratios.

[12] Note that the expression $m_{t+\Delta t} - E_t[m_{t+\Delta t}]$ involves only the innovations to the log P/D ratios.

[13] followed by Ferson, Nallareddy, and Xie (2009)

[14] Ferson, Nallareddy, and Xie (2009) do not use log P/D factors in their analysis so their linear beta pricing relationship is different from (21) in that respect.

This relation, which is a restricted form of the relation (19), makes the approximation that time varying expected returns are unimportant. Hence, we can use the relation between the results of the cross sectional regressions performed on the basis of (19) and (21) to assess the importance of time varying expected returns for the set of assets used in the study.

We ignore the contemporaneous consumption growth factor in the empirical analysis since it is well-known that it does not explain the cross section of equity returns (such an approach is also used, for e.g., by Malloy, Moskowitz, and Vissing-Jørgensen (2009)). We have verified that it's inclusion does not materially affect any of the results.[15]

A. Relation to Standard Linear Factor Models

We note from the standard loglinear approximation of Campbell and Shiller (1988) that innovations of the log P/D ratios are similar to excess returns minus dividend growth. Hence, the methodology involved here closely parallels the standard factor analysis and principal component analysis methods of Connor and Korajczyk (1986), Lehmann and Modest (1988), Lehmann and Modest (2005) and Connor and Korajczyk (2009) but differs in the way the factors are constructed. While standard factor analysis constructs factors from the returns themselves, this methodology pays more attention to returns that are not explained by contemporaneous dividend growth, or in other words, to the more interesting non-trivial part of returns. When the lagged principal components of the log P/D ratios are included as factors as in the full linear beta pricing relationship, it extends factor analysis to include important but slow moving predictable components of excess returns and consumption and dividend growth which are at the heart of long run risk models. Thus, we see that long run risk models can be related to factor models in the literature with the factors including both the innovations of the long run risk

[15]These results are available upon request from the authors.

components (which are analogous to returns) and the components themselves (which are analogous to the price dividend ratios).

We emphasize that the above relation does *not* imply that the excess return factor structure is the same as that of the log P/D ratios. This is because of the presence of a factor structure in the dividend growths of various portfolios. Hence, there is no contradiction between our finding of two factors in the log P/D ratios of the 25 Fama-French portfolios and the well-known result that the excess returns of these portfolios exhibit a three factor structure.

B. P/D or P/E ratios?[16]

A natural question is whether the use of price-earnings (P/E) ratios is preferable to the use of P/D ratios in the empirical analysis. This question arises from the observation that a number of firms do not pay cash dividends and for such firms earnings are generally viewed as a better measure. However, this is not an issue for us as we work with portfolios of stocks.

The primary theoretical reason for our preference for using P/D ratios is that the asset pricing relations we derived earlier makes use of the fact that the stock price is the expected discounted value of future dividends. Hence the asset pricing equations we derived will *not* hold if dividends are replaced by earnings unless the "true" payout ratio (i.e. the payout ratio using the unmeasured true dividends) of firms is unrelated to the risk premium which, in long run risk models, is in general an affine function of the underlying state variables (as shown by Bansal and Yaron (2004), it is an affine function of only the consumption growth volatility in their specification and it is easy to show in a similar manner that this is the case for our specification as well). As noted in the empirical section, we do find that the payout ratio is strongly related to the consumption

[16] We thank seminar participants at INSEAD for bringing up this interesting and important question.

growth volatility and this implies that it is inappropriate to use P/E ratios instead of P/D ratios in our analysis.

Another reason for using P/D instead of P/E ratios is that earnings are regularly revised and that these revisions are strongly related to asset prices as documented by Da and Warachka (2009). The extent of the predictability of these revisions is unclear. Since the information structure of the economy being considered here implies that agents must use estimates of the final revised earnings to compute the P/E ratios, the validity of a test of the model using them (even ignoring the dividend smoothing issue pointed out above) will depend on the validity of the model used for the predictable earnings revisions.

IV. Data

In this section, we describe the data used in this paper. Consumption data is obtained from the National Income and Product Accounts (NIPA) tables available at the BEA web site. Real annual per capita consumption is defined to be the nominal aggregate annual consumption of nondurables and services divided by the NIPA estimate of the mid-year population and deflated by the implicit personal consumption deflator.[17] Annual consumption growth is defined to be the first difference of the logarithm of this series. Quarterly seasonally adjusted consumption data is also obtained from NIPA and it's growth is defined in an analogous manner.

The proxy for the nominal risk-free rate is the Fama 3 month T-bill rate obtained from CRSP. It is converted to three proxies of the real risk free rate using the realized, past and expected inflation as measured by the future CPI growth, lagged CPI growth and expected growth in the CPI (as discussed in the relevant section of this paper). The

[17]Since we make use of data expressed in terms of chained dollars, we use a Tornqvist type index (Whelan 2000) to construct the implicit consumption deflator.

CPI data for the calculation of the first two measures is obtained from CRSP while the expected CPI growth data is obtained from the Federal Reserve Bank of Philadelphia.

The stock market proxy (used to determine the relationship between the factors and future dividend growth and expected returns) is defined as the CRSP value-weighted index of all stocks listed on the NYSE, AMEX and NASDAQ. The construction of portfolios based on size and book-to-market ratios is as in (Fama and French 1993) and (Fama and French 1996). Data on the 6 (2×3) and 25 (5×5) portfolios sorted on the basis of both these characteristics as well as the two sets of ten portfolios (deciles) sorted on either characteristic is obtained from Ken French's web site. In this paper, the growth and value portfolios respectively denote the bottom and top book to market ratio deciles.

For testing the asset pricing relationships with portfolios other than the ones used to estimate the factors (we call this out of sample testing), we use three sets of ten portfolios each formed on the basis of long term reversal, short term reversal and the earnings to price (E/P) ratio. The long term reversal portfolios are formed monthly on the basis of stock's return over the past five years minus it's return over the past year. In other words, they are formed at time $t-1$ (time being indexed by month) by sorting stocks into ten portfolios according to their returns from $t-61$ to $t-13$. Similarly, short term reversal portfolios are formed at time $t-1$ by sorting stocks into ten portfolios based on their return from $t-2$ to $t-1$. The E/P based portfolios are formed at the end of June of year t by sorting stocks into ten portfolios (using NYSE breakpoints) on the basis of their E/P ratios where earnings are earnings before extraordinary items during fiscal year $t-1$ and the price is the market capitalization at the end of December of year $t-1$. Data on these thirty portfolios is also obtained from Ken French's web site.

Monthly dividends of these portfolios are calculated using the difference between the returns of the corresponding dividend reinvested and non-reinvested portfolios. The price-dividend ratios are then calculated by dividing the real price of the non-reinvested portfolio by the sum of the lagged twelve real monthly dividends. This procedure accounts for the pronounced seasonality of the dividend series. The nominal prices and dividends are deflated by the CPI to get these real prices and dividends. As pointed by Van Binsbergen and Koijen (2010), the effect of assumptions regarding the handling of dividends paid during the year on the price dividend ratios is negligible with the correlation between the different measures being about 0.9999.

Real time consumption data is obtained from the web site of the Federal Reserve Bank of St. Louis and is described by Croushore and Stark (2001). The real consumption during a quarter is defined to be the sum of the real consumption of nondurables and services during that quarter. The real consumption during a year is defined to be the sum of the real consumptions during each quarter of that year. Real per capita consumption during a period is defined to be the real consumption during that period divided by the mid-period estimate of population. The real time annual per capita consumption growth for year t is defined to be the difference between the logarithms of the real per capita consumptions during years t and $t-1$ respectively as calculated using data of vintage Q1 of year $t+1$. To provide an example, the data set of Q1 1976 vintage is used to construct the real time annual per capita consumption growth for 1975. It is constructed by adding the real nondurables and services consumptions of Q1-Q4 1974 and Q1-Q4 1975, dividing each of them by the mid-year estimates of the population, and then taking the difference of the logarithms of the corresponding quantities.

V. Empirical Findings

A. Structural Break Implied by the Factors

Marakani (2009) documents strong evidence that the parameters of long run risk models could not have been the same before and after 1942. Hence, we only consider the post

1942 period in our analysis and assume that consumers are myopic and do not consider the possibility of regime change in the model.[18] We defer the examination of an extended model where the consumers are aware of possible regime shifts to future research.[19]

B. Construction of the Principal Components and Their Innovations

From (6), the problem of obtaining the factors of log P/D ratios is, for a fixed number of factors $n+m$, equivalent to the problem of finding time series processes $F_{i,t}^{n+m}$ to solve

$$V(n+m) = \min_{\Lambda, F^{n+m}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(X_{i,t} - \Lambda_i^{n+m} F_{i,t}^{n+m} \right)^2 \tag{22}$$

where X is the matrix of demeaned log P/D ratios, N is the number of portfolios, T is the length of the time series, F are the factors and Λ are the loadings of the individual log P/D ratios on them (the superscript $n+m$ keeps track of the number of assumed factors). The equivalency of the two problems follows trivially from the assumption that the error terms are i.i.d and Gaussian. Hence, this problem is the same as the well studied standard factor analysis problem (of which Connor and Korajczyk (2009) is an excellent review). The assumptions regarding the error terms are not crucial for our results as they hold even if we perform the principal component analysis after first scaling the log P/D ratios to make them each have unit variance or after first scaling them each according to their residual variances. In other words, our results are robust to the use of different specifications for the error term.

Hence, the factors can be calculated by singular value decomposition of the matrix of de-meaned log P/D ratios. This is equivalent to the more usual method of using the

[18] The use of post-1945 or post-1950 data does not significantly change our results.

[19] In this context, we note that Bekaert and Engstrom (2010) have recently argued that habit formation models are better able to incorporate the very different dynamics observed during and after the Great Depression.

eigenvectors of the covariance matrix or directly solving (22), but is preferred because it has greater numerical stability. The number of relevant factors $k = m + n$ is determined by using the information criterion

$$\underset{k}{\operatorname{argmin}} \, \mathrm{IC}_{p2} \equiv \underset{k}{\operatorname{argmin}} \left(\log V(k) + 2k \left(\frac{N+T}{NT} \right) \log \min(N,T) \right) \tag{23}$$

suggested by Bai and Ng (2008) and by Connor and Korajczyk (2009). This method is known to be consistent when the number of quantities and the length of the time series become large. As pointed out by Bai and Ng (2008), traditional methods usually overestimate the number of factors that are present in the data.

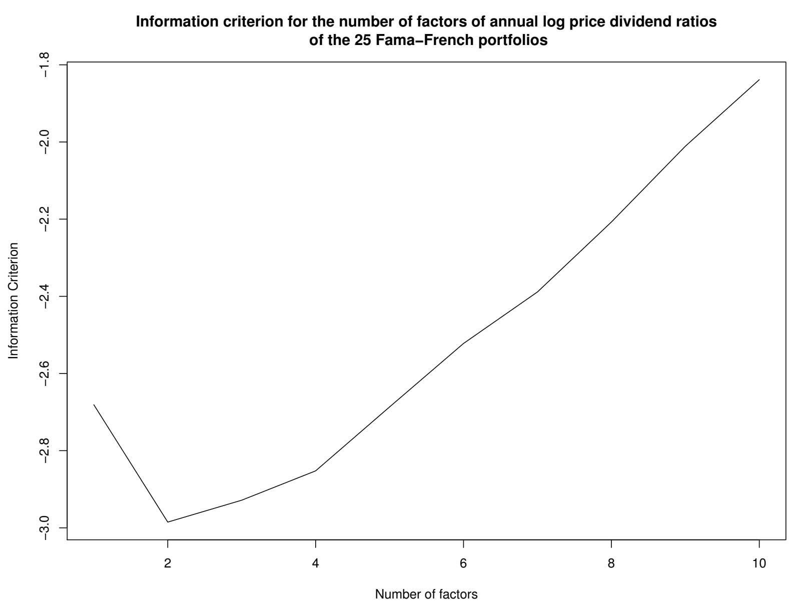

Figure 1
Information criterion as a function of the number of factors for the annual log P/D ratios of the 25 Fama-French portfolios.

We carry out this procedure on the annual and quarterly log P/D ratios of the 25 Fama French portfolios from 1943 (to account for the structural break) and 1947 (since quarterly consumption data is only available from this date) respectively. We find two significant factors in both series (as well in the monthly series of log P/D ratios from

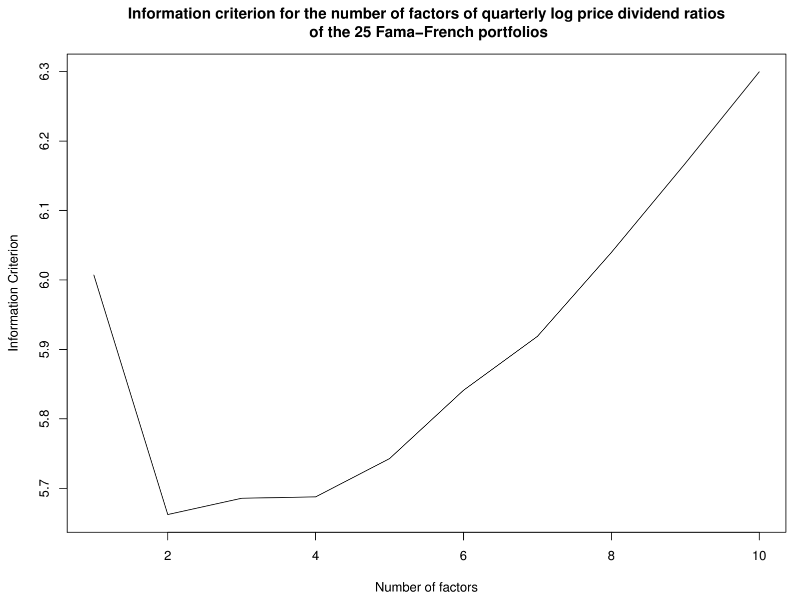

Figure 2
Information criterion as a function of the number of factors for the quarterly log P/D ratios of the 25 Fama-French portfolios.

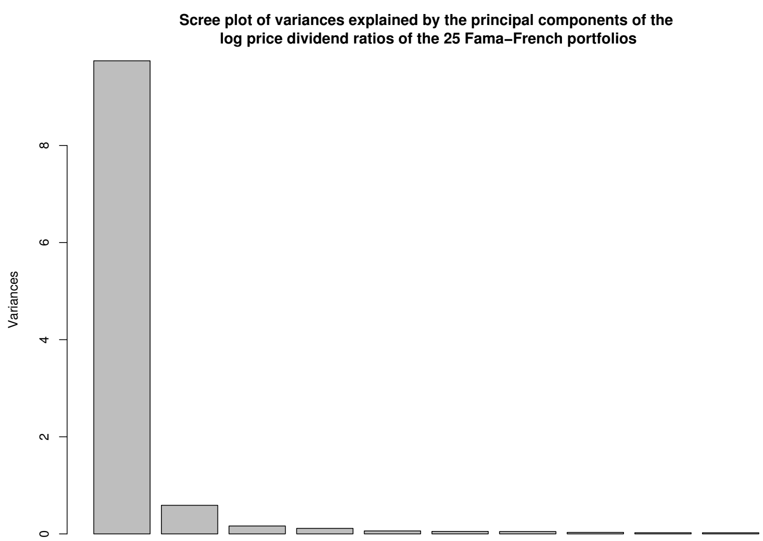

Figure 3
Variances explained by the first ten principal components of the annual log P/D ratios of the 25 Fama-French portfolios.

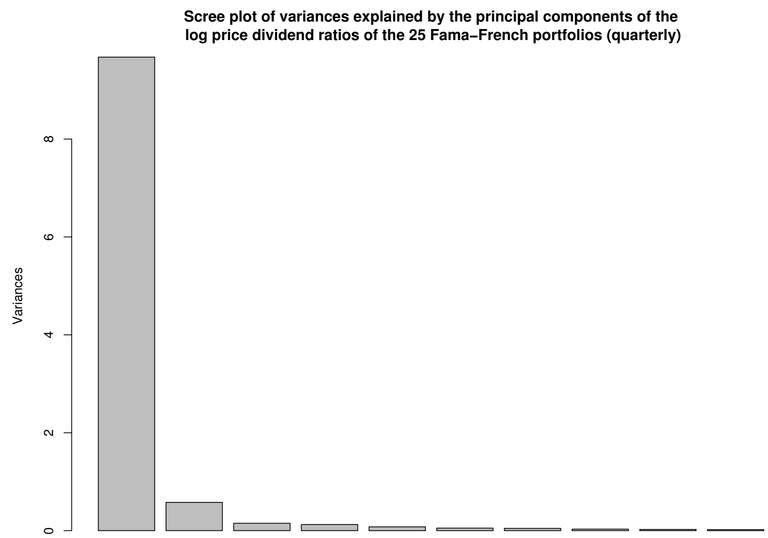

Figure 4

Variances explained by the first ten principal components of the quarterly log P/D ratios of the 25 Fama-French portfolios.

1947).[20] We plot the information criterion as a function of the number of factors in figures 1 and 2, and the variances explained by the principal components in figures 3 and 4 respectively.

Using the same procedure, we find two factors in the first differences of the quarterly log P/D ratios of these portfolios. We also note that the plot of the variances explained by their principal components in figure 5 unambiguously points to a two factor structure.

We tabulate the rotations that relate the annual and quarterly log P/D ratios of the 25 Fama-French portfolios to their first two principal components, denoted by $F_{1,2}^{a,q}$ with the superscript representing the frequency of observation and the subscript the principal component, in table (II). We note that the estimated rotation matrices are essentially independent of the measurement frequency and that most of the small differences in

[20]The date change from 1943 to 1947 makes only a minimal difference to the estimated factors and the subsequent results remain largely unchanged even if the quarterly factors are estimated using data from 1943.

Table II
Rotation matrices that relate the log P/D ratios to their first two principal components

The rotation matrices that relate the log annual and quarterly price dividend ratios of the 25 Fama-French portfolios to their first and second principal components. F_1^a and F_2^a represent the first and second principal components of the annual log P/D ratios while F_1^q and F_2^q represent the first and second principal components of the quarterly log P/D ratios.

Rotation matrix for F_1^a

		Growth				Value
		1	2	3	4	5
Small	1	0·356	0·267	0·206	0·194	0·169
	2	0·354	0·244	0·198	0·168	0·135
	3	0·314	0·210	0·176	0·152	0·135
	4	0·234	0·176	0·154	0·125	0·100
Large	5	0·148	0·116	0·104	0·114	0·116

Rotation matrix for F_2^a

		Growth				Value
		1	2	3	4	5
Small	1	−0·458	−0·140	−0·082	0·028	0·104
	2	−0·289	−0·055	0·089	0·194	0·211
	3	−0·177	0·025	0·185	0·231	0·221
	4	−0·064	0·091	0·170	0·302	0·243
Large	5	−0·005	0·077	0·186	0·275	0·315

Rotation matrix for F_1^q

		Growth				Value
		1	2	3	4	5
Small	1	0·359	0·268	0·204	0·195	0·175
	2	0·348	0·244	0·198	0·170	0·143
	3	0·309	0·210	0·177	0·152	0·140
	4	0·229	0·176	0·154	0·127	0·106
Large	5	0·147	0·116	0·105	0·114	0·113

Rotation matrix for F_2^q

		Growth				Value
		1	2	3	4	5
Small	1	−0·506	−0·155	−0·080	0·019	0·066
	2	−0·265	−0·046	0·092	0·191	0·168
	3	−0·153	0·029	0·192	0·241	0·198
	4	−0·041	0·099	0·168	0·298	0·211
Large	5	0·014	0·086	0·191	0·289	0·323

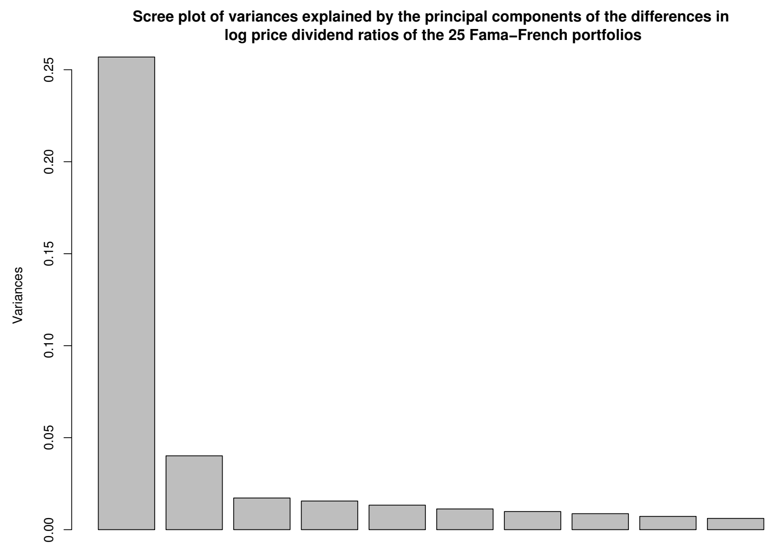

Figure 5

Since a factor structure for the log P/D ratios also implies a similar factor structure for the first differences in the log P/D ratios, we check that the first differences of the log P/D ratios also exhibit a two factor structure in the data. We do so by plotting the variances explained by the first ten principal components of the first differences of the log P/D ratios of the 25 Fama-French portfolios and find that it clearly supports the two factor structure hypothesis.

the two sets of rotation matrices are due to the change in the starting date for the data used in their construction. Hence, where no fear of confusion arises, we ignore the measurement frequency and denote the two principal components by F_1 and F_2 respectively. From the rotation matrices, we find that F_1 loads positively on all the portfolios and loads slightly more on the small stock portfolios. In contrast, F_2 loads positively on large and value stocks and negatively on growth and small stocks. We thus expect F_2 to be closely related to the cross sectional differences among the portfolios.

We estimate the innovations of the two identified principal components as the OLS residuals obtained on regressing them on n lags of themselves, n being the smallest value for which they are serially uncorrelated at the 10% level according to both the Ljung-Box and Durbin-Watson tests. n is always found to be one for the annual data and sometimes two for the quarterly data. We denote these estimated innovations as IF_1^f and IF_2^f with $f = a, q$ representing the measurement frequency.

C. Principal Components & the Long Run Risk Factors

Since the X_i factors represent joint predictable components of consumption and dividend growth, a positive and significant coefficient should result on regressing future consumption and dividend growth against these factors. Similarly, since the V_j factors are components of the consumption growth volatility, regressing consumption growth volatility against them should also lead to a positive and significant coefficient. Since the principal component analysis only identifies affine transformations of the full set of long run risk factors, we can, in general, expect to find that the principal components will be related to both the X_i and V_j factors and that both regressions above will lead to significant coefficients given that the long run risk model holds. However we find that only the volatility regression generates a significant coefficient for the first identified principal component F_1 and that only the future consumption and dividend growth regression generates a significant coefficient for the second identified principal component F_2. This implies that the first identified principal component is naturally identifiable as an affine function of the only V factor and that the second identified principal component is naturally identifiable as an affine function of the only X factor.

We now examine the volatility regression in some detail. In order to construct a consumption volatility series, we estimate the innovations of quarterly consumption growth $\epsilon_{v,t}$ as the OLS residuals obtained on regressing it on n lags of itself, n being the smallest value for which they are uncorrelated at the 10% level according to both the

Ljung-Box and Durbin-Watson tests. n is found to be three for this data series. Using these estimated innovations, we estimate the consumption volatility series as

$$v_t^n = \log \sum_{i=1}^{n} \frac{\epsilon^2_{v,\lfloor t+n/2-i \rfloor}}{n} \tag{24}$$

This methodology is standard and has been used in the context of long run risk models by Beeler and Campbell (2009).

The results of regressing v_t^{24}, v_t^{12} and v_t^6 on F_1^q and F_2^q are summarized in table (III). They show that F_1 is very closely related to consumption growth volatility with the R^2 of the 24 quarter volatility regression being as high as 81%. Even the R^2 for the 6 quarter volatility regression, where measurement error is likely to be high, is quite high at 47%. Further, the fact that the coefficients of F_1 in the various regressions are very similar to each other (i.e. for volatilities estimated over several horizons) provides strong evidence that the relation is robust. In contrast, there is no evidence at all that F_2 is related to consumption growth volatility. This result, when combined with the result, detailed below, that F_1 is unrelated to future consumption and dividend growth, leads to the conclusion that F_1 is an affine function of a V type factor. This conclusion follows because F_1 satisfies the conditions we have identified for such a factor : it is an affine function of log P/D ratios, it tracks consumption growth volatility and does not predict future consumption or dividend growth.

Before we go ahead to examine the consumption and dividend growth regressions, we present evidence for our earlier assertion (during the discussion of the appropriateness of using P/E ratios instead of P/D ratios) that the stock payout ratio is strongly related to the consumption growth volatility factor which, in long run risk models, is affinely related to the equity risk premium as shown by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007). We first plot the overall market payout ratio in figure 6 (the data used to calculate the ratio was obtained from NIPA) and note that it is not close to constant as is required if P/E ratios are to be used instead of P/D ratios in the analysis.

Table III
Regression of consumption growth volatility on the significant principal components of the quarterly log P/D ratios

Result of regressing volatility as defined in (24) against F_1^q and F_2^q, the two significant principal components of the quarterly log P/D ratios. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994).

	F_1^q	F_2^q	R^2
24 quarter volatility	-0.213^{***} (0.027)	0.081 (0.094)	81.2%
12 quarter volatility	-0.234^{***} (0.052)	0.074 (0.190)	62.7%
6 quarter volatility	-0.235^{***} (0.062)	0.024 (0.209)	46.9%

Table IV
Regression of the payout ratio on the lagged principal components

Result of regressing the payout ratio on F_1^a and F_2^a, the two significant principal components of the log P/D ratios. Note that since F_1^a is negatively related to consumption growth volatility, this implies a *negative* relationship between the payout ratio and the latter. The standard errors are Newey-West corrected with the number of lags determined by the procedure in Newey and West (1994)

Intercept	Coefficient of F_1^a	Coefficient of F_2^a	R^2
0.300 (0.005)	0.0216^{***} (0.0028)	-0.014 (0.012)	71.8%

The results of regressing the payout ratio on the observed factors are summarized in table (IV). From it, we see that the payout ratio is strongly related to consumption growth volatility and, therefore, in the context of the long run risk model, to the equity risk premium. This relation implies that it is not possible to use P/E ratios as a proxy for P/D ratios to test this model as the timing of realized cash flows is important within it's context.

We now examine the dividend and consumption growth regressions in detail. The results of regressing annual real market dividend growth (i.e., growth of annual market dividends deflated by the CPI) on the lagged values of F_1^a and F_2^a are summarized in table (V). We find, from them, that F_2^a, but not F_1^a, predicts market dividend growth. This predictive ability is weakly robust to lagging twice to account for time aggrega-

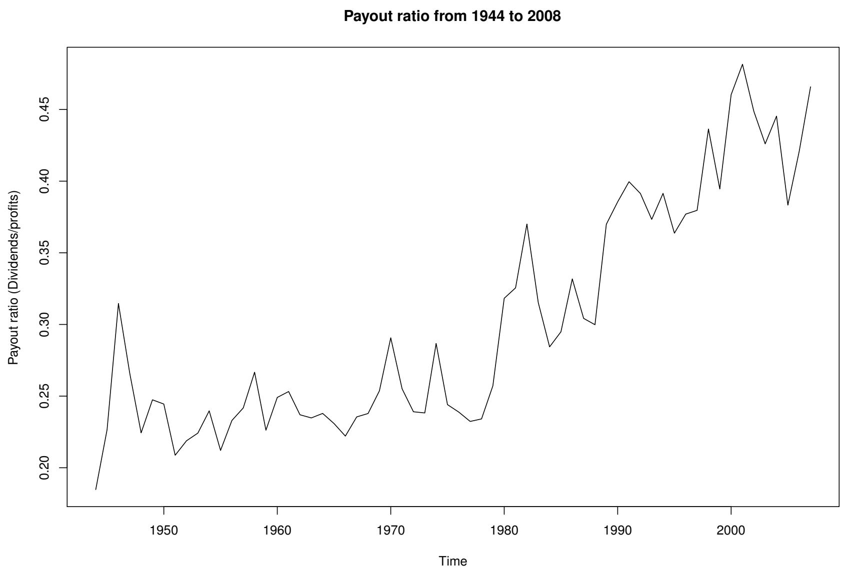

Figure 6
Overall payout ratio in the economy from 1944 to 2008.

tion with the coefficient for F_2^a being significant at the 10% level.[21] (It should also be noted that time aggregation is generally not considered an issue with respect to dividend growth.) While our results are robust to time aggregation of dividend growth along this dimension, we acknowledge that such aggregation also leads to biases in our estimates of the price dividend ratios since we calculate them, as is conventional in the literature, using dividends aggregated on an annual basis in order to adjust for their pronounced seasonality.[22] However, we also note that time aggregation of dividend growth, in contrast to time aggregation of consumption growth, is generally not considered a significant issue in the literature, that our model specification is annual rather than monthly and that our results are robust to the use of either of the conventional assumptions regarding the investment of dividends received during the year (the first being that such dividends

[21] It is interesting that F_2^a, which weights the value portfolios more heavily, predicts future market dividend growth better than the log market P/D ratio (whose inability to predict dividend growth is well known (Cochrane 2005)). We hypothesize that this is because value stocks have a low duration which makes their P/D ratios depend more on dividend growth than on future expected excess returns.

[22] We thank Dana Kiku for bringing this to our attention.

Table V
Regression of market dividend growth and real time consumption growth on the lagged principal components

Results of regressing real annual market dividend growth (d_m) and real time consumption growth (c^{RT}) against lagged F_1^a and F_2^a, the two significant principal components of the annual log P/D ratios, and lagged log market P/D ratio ($\log(P/D)_m$). The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994). The regressions using the log market P/D ratio are for the same time period as for the ones using F_1^a and F_2^a (1944-2008 for the 1 year dividend growth regressions and 1946-2008 for the 3 year dividend growth regressions).

Regression of market dividend growth on the lagged F_1^a and F_2^a, compared with that on the lagged log market P/D ratio
(IPCD stands for the implicit personal consumption deflator)

	$F_{1,t}^a$	$F_{2,t}^a$	$\log(P/D)_{m,t}$	R^2
$\Delta d_{m,t+1}$ deflated by CPI	0.0004 (0.0031)	0.0317*** (0.0079)		16.0%
			0.003 (0.027)	0.0%
After 1 yr	-0.0005 (0.0032)	0.0134* (0.0089)		2.9%
$\Delta d_{m,t+1}$ deflated by IPCD	-0.0002 (0.0032)	0.0288*** (0.0079)		13.3%
			-0.002 (0.026)	0.0%
After 1 yr	-0.0012 (0.0033)	0.0126* (0.0088)		2.9%
$d_{m,t+3} - d_{m,t}$ deflated by CPI	0.0008 (0.0135)	0.0571** (0.0261)		11.6%
			0.005 (0.107)	0.0%
$d_{m,t+3} - d_{m,t}$ deflated by IPCD	-0.0012 (0.0132)	0.0543** (0.0247)		10.9%
			-0.011 (0.103)	0.0%

Regression of real time annual consumption growth on lagged F_1^a and F_2^a

	$F_{1,t}^a$	$F_{2,t}^a$	R^2
Δc_{t+1}^{RT}	$-2 \times 10^{-4}(9 \times 10^{-4})$	0.0068* (0.0037)	17.4%
Δc_{t+2}^{RT}	$-5 \times 10^{-4}(7 \times 10^{-4})$	0.0054** (0.0021)	9.8%
$\Delta c_{t+1}^{RT} + \Delta c_{t+2}^{RT}$	$-5 \times 10^{-4}(1.5 \times 10^{-3})$	0.0123** (0.0050)	18.9%

are invested in nominal cash until the end of the year and the second being that they are invested in the asset itself until the end of the year[23]).

[23]The convention used in the presented calculations is equivalent to the assumption that dividends received during the year are consumed immediately and that the agent is completely indifferent to the timings of these dividends *during* the year.

The results of regressing annual real time consumption growth against the lagged values of F_1^a and F_2^a are also summarized in table (V). We find, from them, that F_2^a also predicts real time consumption growth as defined in the data section and that this predictive ability is robust to lagging twice to account for time aggregation. This is in accordance with the long run risk hypothesis that dividend and consumption growth share the same persistent component(s) X.[24] From the results in table (V), we conclude that F_2 can be identified as an affine function of a X type factor as it satisfies the essential properties of such factors : it is an affine function of log P/D ratios, predicts dividend and consumption growth but not consumption growth volatility.

We find that F_2 also satisfies another expected property of the X factor in many long run risk models. It has been pointed out by Bansal, Yaron, and Kiku (2007), Bansal, Dittmar, and Lundblad (2005), Bansal, Dittmar, and Kiku (2009), Da (2009) and others, the long run risk model implies that assets with higher sensitivity of predictable dividend growth to the long run risk factor X, which is measured by $\phi_{i,l}$ in our model, have higher expected excess returns.[25] Hence, if the long run risk model holds, we will generally expect to find that the coefficients obtained on regressing the future dividend growths of various portfolios against F_2 are significantly different from each other and that they are related to their expected excess returns. We do find that to be the case with the F statistic strongly rejecting the equality of the regression coefficients obtained on regressing real dividend growth of each of the 25 portfolios on the lagged value of F_2 (the value of the statistic being 2.20 ($p < 0.001$)). We also find, as expected, that these regression coefficients are higher for the portfolios of small and value stocks which have

[24] We note that while the use of this measure of consumption is not standard, it is more relevant for the current analysis as it better matches the information structure of the consumers in the economy. (It is also possible that real time data captures the sentiment of consumers as it reflects their current view of the state of the economy.)

[25] While this is not *necessarily* true in our version of the long run risk model as we do not set $\pi_{i,l,x}$ to zero, our version still implies a positive relationship between $\phi_{i,l}$ and the expected excess return of asset l holding $\pi_{i,l,x}$ constant.

higher excess returns. We note that this result is not very surprising given the form of the rotation matrix relating F_2 to the log P/D ratios.[26]

Given the long run risk model, we also expect to find little if any cross-sectional variation of the sensitivity of dividend growth to the volatility factor. Empirically, we do find that this is largely the case with corresponding F statistic for F_1 (the principal component related to the volatility) being much lesser at 1.73 ($p = 0.015$). While this is marginally significant, it is mostly because the regression coefficients for the portfolios corresponding to the smallest stocks being larger than the others.[27] Since the size premium is much less robust than the value premium, we see that this cross-sectional variation is not strongly related to expected excess returns.

We do note that there is no strong relation between the two principal components and short term consumption growth when consumption is defined as the consumption of nondurables and services estimated with the use of current data. However, we also find, from the results of regressing current annual consumption growth against the fourth lag of IF_2^a, which are summarized in table (VI), that there is a significant negative relation between them which could mask the true long run relation between F_2^a and future consumption growth. Keeping this in mind, we examine the results of regressing five year consumption growth after the first five years (i.e. $c_{t+10} - c_{t+5}$) against the lagged principal components which are also summarized in table (VI). We find that the coefficient of F_2 is significant at the 10% level (using Newey-West corrected standard errors) and that the coefficient of F_1 is insignificant. Hence, there is some evidence that F_2 is positively related to future long term consumption growth even when the conventional measure is used. We find that this positive relationship is concentrated in the services sector and that regressing three or five year services consumption growth after the first five years on the lagged principal components gives rise to coefficients which are significant at the

[26]As in Bansal, Dittmar, and Lundblad (2005), most of the individual coefficients are not significant but they are significantly *different* from each other.

[27]We used annual values for the above analysis in order to eliminate issues arising from dividend and CPI seasonality and to minimize the confounding effects that arise from overlapping regressions.

1% level. This result is consistent with the evidence documented by Marakani (2009) that services consumption growth exhibits highly significant long term autocorrelations unlike nondurables consumption growth. Further, the fact that the significant coefficient is that of F_2 and that it is of the same sign as it's regression coefficient for future market and cross-sectional dividend growth constitutes significant evidence that F_2 captures a long run component which is related to both future consumption and dividend growth. We further note that the log market P/D ratio does not predict consumption growth even after a gap of a few years unlike F_2 and that the R^2 of the regression of $c_{t+10} - c_{t+5}$ on the lagged log market P/D ratio is a negligible 10^{-6}.[28] Hence, it does not capture any such long run component as noted by Beeler and Campbell (2009).

Table VI
Regressions examining the relation between conventionally measured consumption growth and the lagged principal components

The first table tabulates the result of regressing annual consumption growth on the fourth lag of IF_2^a, the innovation of the second principal component of the annual log P/D ratios. The second and third tables respectively tabulate the result of regressing five year overall and services consumption growth after five years (i.e., $c_{t+10} - c_{t+5}$ and $c^{ser}_{t+10} - c^{ser}_{t+5}$) on lagged F_1^a and F_2^a, the two significant principal components of the annual log P/D ratios. The standard errors in the latter two tables are Newey-West corrected with the number of lags determined by the procedure in Newey and West (1994).

Regression of annual consumption growth on the fourth lag of IF_2^a

	Intercept	$IF^a_{2,t-3}$	R^2
Δc_{t+1}	0.0194 (0.0015)	-0.0079^{**} (0.0037)	7.3%

Regression of five year consumption growth after five years on lagged F_1^a and F_2^a

	Intercept	$F^a_{1,t}$	$F^a_{2,t}$	R^2
$c_{t+10} - c_{t+5}$	0.104 (0.010)	-0.0015 (0.0039)	0.0164^{*} (0.0090)	19.1%

Regression of five year services consumption growth after five years on lagged F_1^a and F_2^a

	Intercept	$F^a_{1,t}$	$F^a_{2,t}$	R^2
$c^{ser}_{t+10} - c^{ser}_{t+5}$	0.121 (0.011)	-0.0048 (0.0049)	0.0214^{***} (0.0066)	46.4%

[28] Using services consumption growth only increases this R^2 to 1.4%.

While we are of the opinion that real time data provides a better measure of consumption growth in the context of long run risk models, some readers will be concerned at the lack of evidence for a strong relationship between conventionally measured consumption growth and log P/D ratios. We note that the lack of this evidence is not surprising as consumption growth is known to be largely unforecastable in the post WW2 period. In order to address these concerns, we note that the relationship we find using conventionally measured consumption growth, while weak, is much stronger than that obtained when only the log market P/D ratio is used (as is done by Beeler and Campbell (2009)). We also note that prior studies examining the relation between returns or dividends and future consumption growth conclude that such relations exist even when the regression coefficients are not conventionally significant. For example, we note that the regressions that provide evidence for the relation between SMB, HML and future consumption growth by Parker and Julliard (2005) and for the relation between cross sectional dividend growth and consumption growth by Bansal, Dittmar, and Kiku (2009) do not give rise to statistically significant coefficients. Further, we note that, as pointed out by Hansen and Sargent (2007), the predictable component of consumption growth can be small enough to be undetectable by standard statistical tests but still large enough to be economically important and the weak but suggestive relation we find is strong enough to significantly affect asset prices.[29]

Since the above regressions involve the whole sample and are subject to potential forward looking bias,[30] we investigate whether the predictability implications that lead to the identification of the first two principal components as affine functions of the long run risk factors X and V hold out of sample in appendix D, and find that they indeed do so. Hence, we see that our results are at least partially robust to forward looking bias.

[29]This is shown in unreported results of asset pricing tests using the innovations of projections of future consumption growth on F_1 and F_2.

[30]We thank Dana Kiku for bringing this point to our attention.

We now investigate the relation between the innovations of the principal components and the Fama-French factors in order to examine what these factors stand for in the context of the long run risk model. To do so, we summarize the results obtained on regressing IF_1^a and IF_2^a on the annual Fama-French factors in table (VII). We find, from them, that IF_1^a and IF_2^a can be approximately written as $Mkt + SMB$ and $Mkt + HML$ respectively. In other words, we find that, in the framework of this analysis, excess market returns are related to both consumption growth and consumption growth volatility, that SMB is related to consumption growth volatility and that HML is related to future consumption and dividend growth.

Table VII
Relation between the innovations to the principal components and the Fama-French factors

Results of regressing IF_1^a and IF_2^a,the innovations of the two significant principal components of the annual log P/D ratios on the Fama-French factors.

	Intercept	$R_m - R_f$	SMB	HML	R^2
IF_1^a	-0.39^{***} (0.09)	3.95^{***} (0.42)	2.10^{***} (0.58)	0.50 (0.56)	68.8%
IF_2^a	-0.19^{***} (0.04)	1.42^{***} (0.18)	-0.22 (0.25)	1.42^{***} (0.25)	60.0%

Given the form of the factors above, we relabel F_1 as F_{-Vol} (the negative sign is to remind us that this the relationship between F_1 and consumption growth volatility is negative) and F_2 as F_X for the rest of this paper. We will also refer to F_{-Vol} as a negative volatility factor and to F_X as a consumption/dividend growth factor.

D. Asset Pricing Tests

In the asset pricing tests below, we ignore the contemporaneous consumption growth factor which is technically required for completeness due to the presence of the $c_{t+\Delta t} - c_t$ term in (9). This is mainly due to two reasons. The first is that there is significant measurement error in consumption growth as we show in appendix B. The second is that it is difficult, if not impossible, to account for the complications introduced by time aggregation which have been discussed in detail by Marakani (2009) and others. We note that these two reasons may partially account for the fact that previous studies have shown that contemporaneous consumption growth is incapable of explaining excess asset returns and that including the contemporaneous consumption growth factor makes no difference to our conclusions.[31] We further note that a similar procedure is followed by Malloy, Moskowitz, and Vissing-Jørgensen (2009).

Since the dividends in this analysis have to be measured annually due to seasonality considerations, the asset pricing restrictions are only strictly correct at the annual time scale. Hence, we restrict ourselves to annual data in the following.

D.1. Cross Sectional Regressions

As explained in the previous section, we examine the results of analyzing both the full and restricted beta pricing relationships (19) and (21). We first present our analysis of the restricted linear beta pricing relationship (21).

Our empirical analysis of the restricted beta pricing relationship (21) (ignoring the first term as explained earlier) reveals that it performs well. This conclusion follows from the results of the OLS and WLS cross sectional regressions for this beta pricing relationship (using IF_{-Vol} and IF_X) which are summarized in table (VIII). We find, both from the table and from figure 7, that the cross sectional performance of the model is fairly good with the OLS R^2 of the cross sectional regression being 64.8%. In addition, we find that $\lambda_{IF_{-Vol}}$ and λ_{IF_X}, the coefficients of $\beta_{IF_{-Vol}}$ and β_{IF_X} in the cross sectional regression, are jointly significantly different from zero at the 5% level (p=0.028). Further, the intercept term in the cross sectional regression is not statistically different from zero.[32]

[31] The results with the contemporaneous consumption growth factor are available upon request.

[32] In unreported results which are available upon request, we find that this is also true for innovations of the principal components at the monthly frequency.

Table VIII
Results of the cross sectional regression $\mathbf{E}[\mathbf{r}_{\mathbf{i},\mathbf{t}+\boldsymbol{\Delta}\mathbf{t}} - \mathbf{r}_{\mathbf{f},\mathbf{t}}] + \frac{1}{2}\text{Var}[\mathbf{r}_{\mathbf{i},\mathbf{t}+\boldsymbol{\Delta}\mathbf{t}} - \mathbf{r}_{\mathbf{f},\mathbf{t}}] \approx \beta_{\boldsymbol{\Delta}\mathbf{c}}\lambda_{\boldsymbol{\Delta}\mathbf{c}} + \sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{n}+\mathbf{m}} \beta_{\mathbf{IF}_\mathbf{j}}\lambda_{\mathbf{IF}_\mathbf{j}}$ for the 25 Fama-French portfolios

Results of the two pass cross sectional regression, together with the corresponding dispersion in βs and pricing errors, of the 25 Fama-French portfolios on IF_{-Vol} and IF_X, the innovations to the negative volatility and consumption/dividend growth factors. For the OLS coefficients, the t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the t values with the correction are reported. The R^2 adjusted for the number of variables is reported below the unadjusted R^2.

	Intercept	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	R^2
OLS	-0.026	0.498	0.432	64.8%
	(-0.77)	(2.17)	(3.77)	(61.6%)
	(-0.52)	(1.60)	(2.67)	
WLS	-0.032	0.586	0.407	
	(-0.67)	(1.92)	(2.64)	

Dispersion in $\beta_{\mathbf{IF}_{-\mathbf{Vol}}}$

		Growth 1	2	3	4	Value 5
Small	1	0.226 (0.033)	0.211 (0.023)	0.169 (0.021)	0.155 (0.018)	0.159 (0.022)
	2	0.190 (0.025)	0.150 (0.018)	0.144 (0.017)	0.141 (0.015)	0.124 (0.020)
	3	0.167 (0.023)	0.130 (0.016)	0.109 (0.015)	0.119 (0.016)	0.113 (0.018)
	4	0.144 (0.020)	0.107 (0.016)	0.111 (0.015)	0.110 (0.015)	0.134 (0.020)
Large	5	0.112 (0.019)	0.084 (0.014)	0.076 (0.017)	0.080 (0.017)	0.089 (0.019)
F-stat = 5.55 ($p < 10^{-16}$)						

Dispersion in $\beta_{\mathbf{IF}_\mathbf{X}}$

		Growth 1	2	3	4	Value 5
Small	1	0.007 (0.087)	0.085 (0.060)	0.102 (0.053)	0.139 (0.048)	0.176 (0.058)
	2	0.044 (0.065)	0.084 (0.047)	0.109 (0.044)	0.158 (0.040)	0.201 (0.051)
	3	0.031 (0.058)	0.131 (0.041)	0.156 (0.039)	0.193 (0.042)	0.200 (0.047)
	4	0.033 (0.051)	0.159 (0.040)	0.186 (0.042)	0.208 (0.040)	0.209 (0.051)
Large	5	0.065 (0.048)	0.143 (0.037)	0.151 (0.044)	0.202 (0.045)	0.251 (0.049)
F-stat = 2.18 ($p = 8.1 \times 10^{-4}$)						

Pricing errors $\times 100$

		Growth 1	2	3	4	Value 5
Small	1	2·80	0·56	−0·81	−2·13	−2·38
	2	2·32	−0·82	−2·25	−0·78	−1·92
	3	0·29	−0·10	−0·71	0·03	−1·40
	4	−1·33	1·66	0·78	1·01	0·76
Large	5	−0·81	0·96	−0·34	1·63	2·95

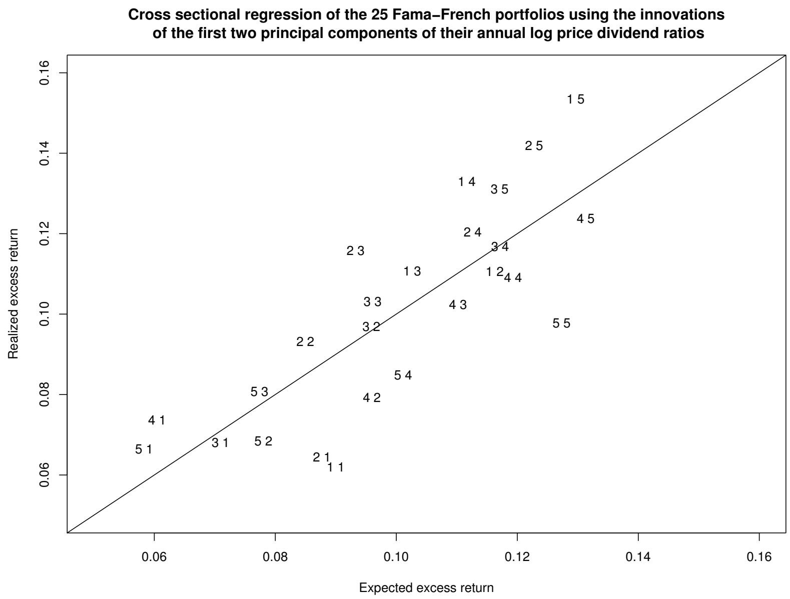

Figure 7

Results of the cross sectional regression of the 25 Fama-French portfolios using IF_{-Vol} and IF_X, the innovations to the negative volatility and consumption/dividend growth factors.

A common concern when using the cross sectional regression methodology is that the betas do not show sufficient cross sectional variation. However, the results summarized in table (VIII) show that this is not the case here and the F test for the hypothesis that the portfolio betas are all equal (for either IF_{-Vol} or IF_X) strongly rejects it ($p < 10^{-3}$).

While the model prices the 25 Fama-French portfolios, which have posed a challenge to traditional consumption based asset pricing models, quite well, it is important to examine whether it is also able to price other portfolios as our factor estimates are derived using them. Hence, we repeat the analysis using three sets of ten portfolios each formed on the basis of long term reversal, short term reversal and the E/P ratio as described in the data section (leaving F_X and F_{-Vol} the same as they have already been identified).

We find that these thirty portfolios are priced well by IF_{-Vol} and IF_X. The cross-sectional regression intercept is negligible (-0.007) and the two innovations are jointly

Table IX
Results of the cross sectional regression
$$\mathbf{E}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] + \frac{1}{2}\mathrm{Var}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] \approx \beta_{\mathbf{\Delta c}}\lambda_{\mathbf{\Delta c}} + \sum_{\mathbf{j=1}}^{\mathbf{n+m}} \beta_{\mathbf{IF_j}}\lambda_{\mathbf{IF_j}}$$
for 30 portfolios

Results of the two pass cross sectional regression, including pricing errors, of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the E/P ratio) on IF_{-Vol} and IF_X, the innovations to the negative volatility and consumption/dividend growth factors. For the OLS coefficients, the t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the t values with the correction are reported. The R^2 adjusted for the number of variables is reported below the unadjusted R^2.

	Intercept	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	R^2
OLS	-0.007	0.292	0.419	72.7%
	(-0.31)	(1.40)	(4.21)	(70.7%)
	(-0.21)	(1.07)	(3.13)	
WLS	-0.016	0.375	0.415	
	(-0.47)	(1.48)	(3.32)	

Pricing errors $\times 100$

	bottom 1	2	3	4	5	6	7	8	9	top 10
Long term reversal	−0·64	−1·88	−0·74	0·09	0·28	−0·93	−0·78	−0·11	1·81	0·16
Short term reversal	2·32	−2·28	−1·61	−0·31	−0·08	1·58	0·86	0·50	0·34	2·53
E/P ratio	−0·70	0·52	0·62	0·60	0·00	−0·65	−1·29	0·52	−0·40	−0·29

significantly priced at the 1% level. The OLS R^2 is found to be 73%. We summarize these results in table (IX) and graph the cross sectional regression in figure 8.

Hence, we conclude that the innovations of the long run risk components (identified from the first two principal components of the log P/D ratios of the 25 Fama-French portfolios) are able to price a variety of equity portfolios which have posed a challenge to traditional asset pricing models.

The above results show that the data supports the restricted linear beta pricing relationship (21) quite well. However, as noted in our earlier discussion, the approximation involved in deriving this relationship is equivalent to the approximation that time varying expected returns are unimportant for the set of assets being considered. Hence,

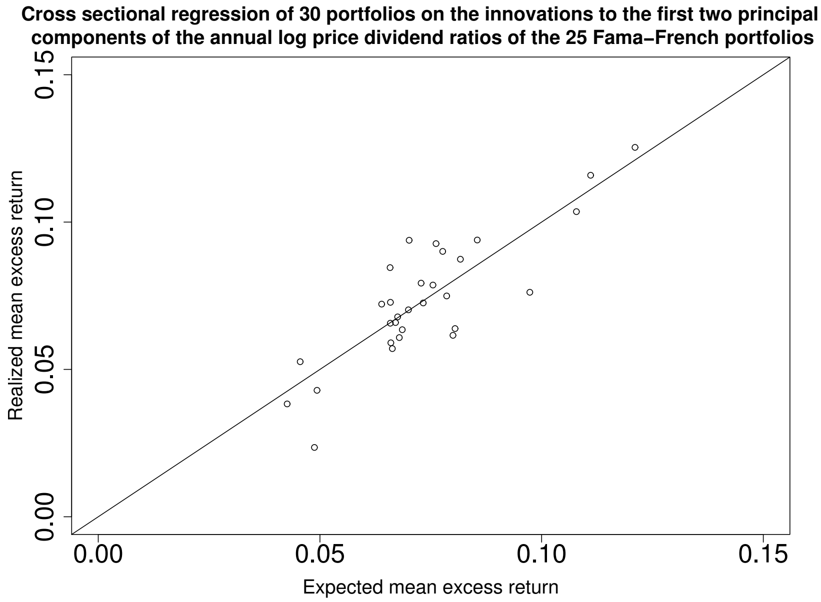

Figure 8

Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the E/P ratio) using IF_{-Vol} and IF_X, the innovations to the negative volatility and consumption/dividend growth factors.

we now investigate the full linear beta pricing relationship (19), which does account for time varying expected returns. We find that it produces even better results for the 25 Fama-French portfolios and similar results for the set of thirty portfolios discussed above. This is seen, for the former, in the cross sectional regression results tabulated in table (X) and plotted in figure 9 and, for the latter, in the cross sectional regression results tabulated in table (XI) and plotted in figure 10. The OLS R^2 is quite high at 77.5% for both cross sectional regressions.[33] Further, the estimates of the cross-sectional regression intercepts are very low for the regressions involving the 25 Fama-French portfolios and the alternate set of thirty portfolios, being 0.3% and -1.1% per year respectively. This

[33]The pricing relationship (19) is also empirically supported at the quarterly frequency. While we do not report detailed results at this frequency for brevity, we note that the cross sectional regression intercept is not significantly different from zero and that the OLS R^2 is greater than 65% for both the 25 Fama-French portfolios and the other set of thirty portfolios.

Table X
Results of the cross sectional regression
$\mathbf{E}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] + \frac{1}{2}\mathrm{Var}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] \approx \beta_{\mathbf{\Delta c}}\lambda_{\mathbf{\Delta c}} + \sum_{\mathbf{j=1}}^{\mathbf{n+m}} \beta_{\mathbf{F_j}}\lambda_{\mathbf{F_j}} + \sum_{\mathbf{j=1}}^{\mathbf{n+m}} \beta_{\mathbf{IF_j}}\lambda_{\mathbf{IF_j}}$
for the 25 Fama-French portfolios

Results of the two pass cross sectional regression, including pricing errors, of the 25 Fama-French portfolios on lagged F_{-Vol} and F_X and concurrent IF_{-Vol} and IF_X. Recall that F_{-Vol} and F_X are the negative volatility and consumption/dividend growth factors and IF_{-Vol} and IF_X are their innovations. For the OLS coefficients, the t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the t values with the correction are reported. The R^2 adjusted for the number of variables is reported below the unadjusted R^2.

	Intercept	$\lambda_{F_{-Vol}}$	λ_{F_X}	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	R^2
OLS	0.003	1.35	-0.568	0.693	0.306	77.5%
	(0.10)	(1.71)	(-3.05)	(2.85)	(2.71)	(73.0%)
	(0.07)	(1.20)	(-2.17)	(2.03)	(1.88)	
WLS	-0.003	1.09	-0.511	0.697	0.306	
	(-0.07)	(1.03)	(-2.13)	(2.10)	(1.99)	

Pricing errors $\times 100$

		Growth				Value
		1	2	3	4	5
Small	1	0·31	1·35	0·53	−0·74	−2·37
	2	1·73	−0·75	−1·38	0·59	−2·81
	3	−0·25	0·03	−1·14	0·00	−0·61
	4	−0·67	1·05	0·66	0·92	0·63
Large	5	−0·32	1·84	−1·37	1·19	1·57

provides important support for our specification as these intercepts must be zero for a correctly specified model as emphasized by Jagannathan and Wang (2007).

The results also imply that, in our framework, time varying expected returns are important to take into account when pricing the 25 Fama-French portfolios but not when pricing the other set of thirty portfolios as the factor risk premia of the lagged factors are only significant for the former.

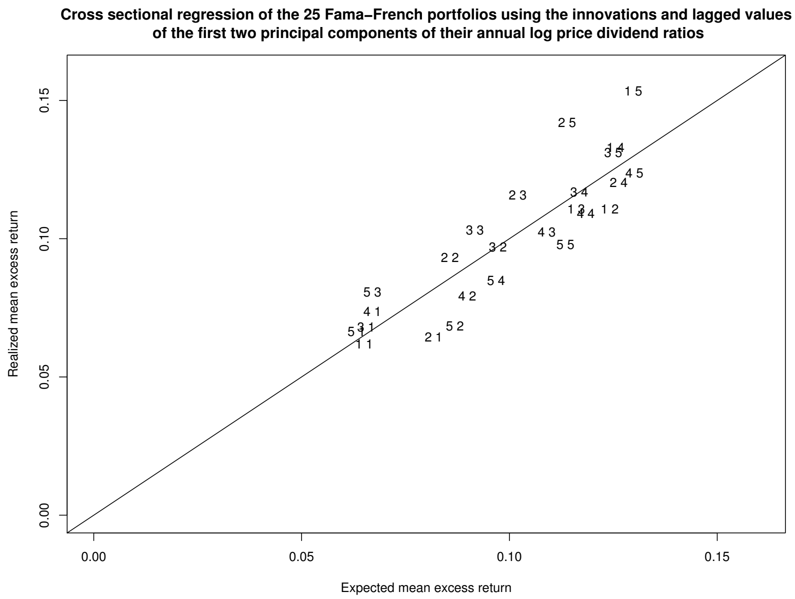

Figure 9

Results of the cross sectional regression of the 25 Fama-French portfolios using IF_{-Vol}, IF_X and lagged values of F_{-Vol} and F_X. Recall that F_{-Vol} and F_X are the negative volatility and consumption/dividend growth factors and IF_{-Vol} and IF_X are their innovations.

D.2. Robustness Tests

Since the excess returns of the 25 Fama-French portfolios formed on the basis of size and book to market ratio have a strong factor structure, it is important to use robust test statistics to eliminate the problem of useless factors being identified as useful (a problem forcefully brought out by Kleibergen (2009) and Kleibergen (2010)). Hence, we test the above cross sectional regressions using the robust test statistics suggested by Kleibergen (2009) in appendix D to ensure that the factors here are not useless. As shown in detail in this appendix, we find that the factors we have identified are not useless according to this test.

We also note that the number of time series observations in our analysis is small due to the low frequency data that we use and that we find the betas of the assets to be significantly different from each other. As noted by Kan and Zhang (1999), these

Table XI
Results of the cross sectional regression
$\mathbf{E}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] + \frac{1}{2}\mathrm{Var}[\mathbf{r_{i,t+\Delta t}} - \mathbf{r_{f,t}}] \approx \beta_{\mathbf{\Delta c}}\lambda_{\mathbf{\Delta c}} + \sum_{\mathbf{j=1}}^{\mathbf{n+m}} \beta_{\mathbf{F_j}}\lambda_{\mathbf{F_j}} + \sum_{\mathbf{j=1}}^{\mathbf{n+m}} \beta_{\mathbf{IF_j}}\lambda_{\mathbf{IF_j}}$
for 30 portfolios

Results of the cross sectional regression, including pricing errors, of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the E/P ratio) on lagged F_{-Vol} and F_X and concurrent IF_{-Vol} and IF_X. Recall that F_{-Vol} and F_X are the negative volatility and consumption/dividend growth factors and IF_{-Vol} and IF_X are their innovations. For the OLS coefficients, the t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the t values with the correction are reported. The R^2 adjusted for the number of variables is reported below the unadjusted R^2.

	Intercept	$\lambda_{F_{-Vol}}$	λ_{F_X}	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	R^2
OLS	-0.011	1.21	-0.038	0.392	0.449	77.5%
	(-0.45)	(1.73)	(-0.14)	(1.74)	(4.73)	(73.9%)
	(-0.29)	(1.24)	(-0.09)	(1.25)	(3.38)	
WLS	-0.018	1.01	-0.056	0.446	0.445	
	(-0.52)	(1.12)	(-0.15)	(1.56)	(3.56)	

Pricing errors $\times 100$

	bottom 1	2	3	4	5	6	7	8	9	top 10
Long term reversal	0·73	−0·89	−0·12	0·48	0·69	−0·61	−0·80	−0·15	1·77	−0·08
Short term reversal	1·30	−2·29	−1·57	−0·03	0·01	1·47	0·71	0·07	−0·39	2·41
E/P ratio	−1·05	0·39	0·75	0·47	−0·22	−1·07	−1·20	0·59	−0·97	−0·41

characteristics make it much less likely that a useless factor is spuriously found to be "useful" in a cross-sectional regression. Finally, we note that the cross-sectional regression intercept should be zero if the model is correctly specified, as emphasized by Jagannathan and Wang (2007), and that the intercept that we obtain for each of the four cross-sectional regressions examined in the previous subsection is indeed close to zero and insignificantly different from it. In particular, this intercept is only 0.3% per year for the cross-sectional regression of the 25 Fama-French portfolios on the full set of variables in the model, i.e. the lagged factors and their innovations.

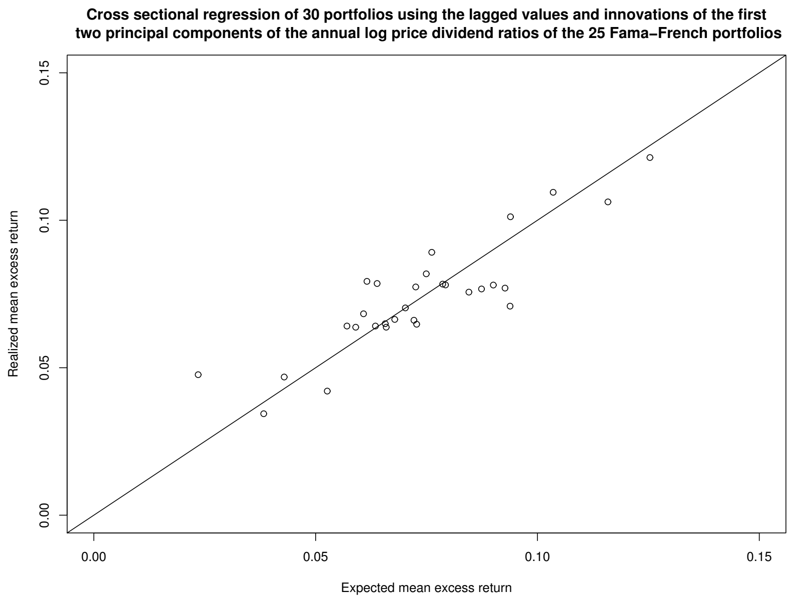

Figure 10

Results of the cross sectional regression of 30 portfolios (three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the E/P ratio) using lagged F_{-Vol} and F_X as well as IF_{-Vol} and IF_X. F_{-Vol} and F_X are the negative volatility and consumption/dividend growth factors and IF_{-Vol} and IF_X are their innovations.

D.3. Relation to the Fama-French Three Factor Model

We now investigate the relationship between the long run risk model as analyzed by us and the standard Fama-French three factor model. Since the intercept for the cross-sectional regression should be zero for a correctly specified asset pricing model, we investigate the results of the constrained cross-sectional regression using the lagged long run risk factors, the long run risk factor innovations and the Fama-French factors (Mkt, SMB and HML) for both the 25 Fama-French portfolios formed on the basis of size and book to market ratio as well as the three sets of ten portfolios formed on the basis of long term reversal, short term reversal and the earnings to price ratio. The results of these cross-sectional regressions are summarized in table (XII).

Table XII
Results of the constrained cross sectional regression including the Fama-French factors for the two sets of portfolios

Results of the constrained cross sectional regression of the 25 Fama-French portfolios and three sets of ten portfolios formed on the basis of long term reversal, short term reversal and E/P ratio on the lagged long run risk factors F_{-Vol} and F_X, the long run risk factor innovations IF_{-Vol} and IF_X and the Fama-French factors Mkt, SMB and HML. The t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first).

Results of the constrained cross sectional regression for the the 25 Fama-French portfolios on the Fama-French factors, lagged F_{-Vol}, F_X and concurrent IF_{-Vol}, IF_X

	$\lambda_{F_{-Vol}}$	λ_{F_X}	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	λ_{Mkt}	λ_{SMB}	λ_{HML}
OLS	1.04	-0.40	0.60	0.31	7.83	2.73	6.43
	(1.63)	(-2.19)	(3.62)	(3.19)	(3.35)	(1.63)	(3.84)
	(1.29)	(-1.69)	(3.07)	(2.44)	(3.31)	(1.62)	(3.77)

Results of the constrained cross sectional regression for 30 portfolios on the Fama-French factors, IF_{-Vol}, IF_X and lagged F_{-Vol} and F_X

	$\lambda_{F_{-Vol}}$	λ_{F_X}	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	λ_{Mkt}	λ_{SMB}	λ_{HML}
OLS	0.33	0.17	0.54	0.32	7.27	0.56	6.67
	(0.51)	(0.79)	(3.54)	(4.16)	(2.88)	(0.22)	(2.95)
	(0.41)	(0.60)	(3.18)	(3.41)	(2.85)	(0.18)	(2.54)

From the tables, we see that while the lagged long run risk factors and the long run risk factor innovations do not drive out the Fama-French factors in the cross-sectional regression, the Fama-French factors also do not drive out the lagged long run risk factors and the long run risk factor innovations. We hypothesize that this could be due to two possible reasons. The first possible reason is that there is measurement error in the estimated long run risk factors. This is highly plausible as the long run risk factors are estimated using P/D ratios and dividend smoothing causes measurement error in the P/D ratios. This means that while relatively slow variations of the long run risk factors can be estimated relatively precisely, their short run variations cannot. These short run variations might be better picked up by the Fama-French factors since they are estimated using returns. The second possible reason is that the Fama-French factors, being determined by returns, also captures liquidity effects which our model does not aim to do.

D.4. GMM Tests

While the cross sectional regression methodology above provides a nice, intuitive way of understanding the importance of the different variables in the stochastic discount factor, it can only handle linear relationships and needs relatively restrictive assumptions for accurate results as pointed out by Jagannathan and Wang (2002). Since the exact Euler equation restrictions

$$E_t[M_{t+1}R^e_{t+1}] = 0 \tag{25}$$

are nonlinear in nature, we now use GMM to ensure that the above results are robust. The results of the GMM estimation of (25) using the 25 Fama-French portfolios sorted on the basis of size and book to market ratio are summarized in table (XIII). The corresponding results for the three sets of ten portfolios sorted on the basis of long term reversal, short term reversal and E/P ratio are summarized in table (XIV).

Table XIII
Results of the GMM test of the Euler equation restrictions
$E[M_{t+1}R^e_{t+1}] = 0$ **for the 25 Fama-French portfolios**

GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ for the 25 Fama-French portfolios sorted on the basis of size and book to market ratio without taking into account the effect of the mean of the stochastic discount factor as suggested by Kan and Robotti (2008).

Identity weighting matrix		**Optimal weighting matrix**	
Coefficient	Estimate (Std. Err.)	Coefficient	Estimate (Std. Err.)
$\Gamma_1\left(\frac{X}{F_X}\right)$	0.463 (0.683)	$\Gamma_1\left(\frac{X}{F_X}\right)$	0.433 (0.282)
$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.056 (0.107)	$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.036 (0.069)
$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	1.734 (0.742)	$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	1.559 (0.448)
$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.114 (0.236)	$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.106 (0.149)
Dist statistic	0.1086	J statistic	23.97 (p=0.29)

Table XIV
Results of the GMM test of the Euler equation restrictions
$E[M_{t+1}R^e_{t+1}] = 0$ **for thirty portfolios**

GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ for the three sets of ten portfolios sorted on the basis of long term reversal, short term reversal and E/P ratio without taking into account the effect of the mean of the stochastic discount factor as suggested by Kan and Robotti (2008).

Identity weighting matrix		Optimal weighting matrix	
Coefficient	Estimate (Std. Err.)	Coefficient	Estimate (Std. Err.)
$\Gamma_1\left(\frac{X}{F_X}\right)$	0.329 (0.480)	$\Gamma_1\left(\frac{X}{F_X}\right)$	0.254 (0.295)
$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.058 (0.117)	$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.075 (0.068)
$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	2.139 (0.884)	$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	1.758 (0.524)
$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.050 (0.286)	$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.004 (0.181)
Dist statistic	0.0603	J statistic	25.43 (p=0.49)

Since excess returns are used in these tests, the mean of the stochastic discount factor must be accounted for by adding an additional moment condition as pointed out by Kan and Robotti (2008). We report the results obtained after adding this moment condition in tables (XV) and (XVI) respectively. These results are found to be closer to those obtained using the cross sectional regression approach. In particular, Γ_1 is found to be fairly large and significant and α_v is found to be negative (note that $\frac{V-\bar{V}}{F_{-Vol}}$ is negative so that a positive coefficient for $\alpha_{IF_{-Vol}}$ implies a negative coefficient for α_v) for the 25 Fama-French portfolios.

Notably, α_x, the market price of risk for shocks to expected consumption growth, is highly significantly positive in all of the GMM estimations with estimates of it's scaled value ranging from 1.56 to 2.90. Since α_x is directly related to the parameter $\gamma - 1/\psi$ which governs the temporal resolution of uncertainty, we can use these estimates to obtain an estimate for it. After making suitable assumptions about the value of ψ, it will also enable us to obtain an estimate for γ.

We note that the identification of the two factors in this study also enables the determination of the relative importance of cash flow and discount rate risks for cross-

Table XV
Results of the GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ together with a moment condition for the mean of M for the 25 Fama-French portfolios

GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ together with an additional moment condition for the 25 Fama-French portfolios to ensure that the biases introduced due to the unspecified mean of the stochastic discount factor are taken into account as suggested by Kan and Robotti (2008). Note that the Dist statistic is *not* comparable with that in table (XIII) as this test has one more restriction. The results are for data from 1944-2007 rather than 1944-2008 as the covariance matrix was very close to singular in the latter data period.

Identity weighting matrix		**Optimal weighting matrix**	
Coefficient	Estimate (Std. Err.)	Coefficient	Estimate (Std. Err.)
$\Gamma_1\left(\frac{X}{F_X}\right)$	1.087 (0.804)	$\Gamma_1\left(\frac{X}{F_X}\right)$	1.031 (0.311)
$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.162 (0.177)	$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.155 (0.107)
$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	2.176 (1.078)	$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	2.204 (0.233)
$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.213 (0.356)	$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	0.214 (0.071)
Dist statistic	0.254	J statistic	9.24 (p=0.99)

sectional returns in the context of long run risk models. This is because the rate at which future equity cash flows are discounted (the equity risk premium) is determined by the consumption growth volatility in these models as shown by Bansal and Yaron (2004) and others, which in turn means that the first factor proxies for discount rate risk and that the second proxies for cash flow risk. The results of the analysis using the innovations of the two factors indicate that cash flow risk is cross-sectionally more important than discount rate risk. This result is robust to the inclusion of the lagged factors, as is seen from the GMM results summarized below. This study thus underlines the importance of cash flow risk and contributes to the recent strand of literature that demonstrates that it can explain a large proportion of the cross-sectional return variation (Campbell and Vuolteenaho 2004) (Bansal, Dittmar, and Lundblad 2005) (Cohen, Polk, and Vuolteenaho 2008) (Campbell, Polk, and Vuolteenaho 2009) (Da and Warachka 2009).

Table XVI
Results of the GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ together with a moment condition for the mean of M for thirty portfolios

GMM test of the Euler equation restrictions $E[M_{t+1}R^e_{t+1}] = 0$ for the three sets of ten portfolios sorted on the basis of long term reversal, short term reversal and E/P ratio together with an additional moment condition to ensure that the biases introduced due to the unspecified mean of the stochastic discount factor are taken into account as suggested by Kan and Robotti (2008). Note that the Dist statistic is *not* comparable with that in table (XIV) as this test has one more restriction.

Identity weighting matrix		**Optimal weighting matrix**	
Coefficient	Estimate (Std. Err.)	Coefficient	Estimate (Std. Err.)
$\Gamma_1\left(\frac{X}{F_X}\right)$	1.164 (0.761)	$\Gamma_1\left(\frac{X}{F_X}\right)$	1.165 (0.420)
$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.002 (0.120)	$\Gamma_2\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.008 (0.115)
$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	2.882 (1.133)	$\alpha_{IF_X} = \alpha_x\left(\frac{X}{F_X}\right)$	2.899 (0.398)
$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.241 (0.301)	$\alpha_{IF_{-Vol}} = \alpha_v\left(\frac{V-\bar{V}}{F_{-Vol}}\right)$	-0.257 (0.135)
Dist statistic	0.228	J statistic	21.37 (p=0.72)

E. Q4-Q4 Consumption Growth and the Long Run Risk Model

Jagannathan and Wang (2007) and Jagannathan, Marakani, Takehara, and Wang (2011) find that Q4-Q4 consumption growth explains the cross section of stock returns much better than annual consumption growth. While these studies interpret the result as being due to infrequent trading by investors, we find that the long run risk model can provide a possible alternative explanation for this result. This is because we find that the correlations between Q4-Q4 consumption growth and the innovations to the long run risk factors IF_{-Vol} and IF_X are much higher than those between annual consumption growth and IF_{-Vol} and IF_X. These correlations, together with the correlations between Qi-Qi consumption growth (where i is 1, 2 or 3) and IF_{-Vol} and IF_X, are summarized in table (XVII)

Using this observation, we conjecture that Q4-Q4 consumption growth may also be serving as a proxy for these innovations to the long run risk factors in the pricing relation. This conjecture is further supported by the fact that Q4-Q4 consumption

Table XVII
Correlations between consumption growths and IF_{-Vol} and IF_X

Correlations between Qi-Qi consumption growth, annual consumption growth and IF_{-Vol} and IF_X (the innovations to the negative volatility and consumption/dividend growth factors).

Consumption growth	IF_{-Vol}	IF_X
Q1-Q1	0.434	0.418
Q2-Q2	-0.301	-0.100
Q3-Q3	-0.083	0.123
Q4-Q4	0.259	0.331
Annual	0.003	0.090

growth is driven out of the cross-sectional regression of the 25 Fama-French portfolios formed on the basis of size and the book to market ratio by IF_{-Vol} and IF_X. The results of this cross-sectional regression are summarized in table (XVIII).

Table XVIII
Results of the cross sectional regression for the 25 Fama-French portfolios using Q4-Q4 consumption growth, IF_{-Vol} and IF_X

Results of the cross sectional regression of the 25 Fama-French portfolios formed on the basis of size and book to market ratio using Q4-Q4 consumption growth, IF_{-Vol} and IF_X (the innovations to the negative volatility and consumption/dividend growth factors) from 1953-2007. For the OLS coefficients, the t values with and without the Shanken correction (Shanken 1992) (Shanken and Zhou 2007) are reported below the coefficient (the value without the correction is reported first) while for the WLS coefficients, only the t values with the correction are reported. The R^2 adjusted for the number of variables is reported below the unadjusted R^2.

	Intercept	λ_{Q4-Q4}	$\lambda_{IF_{-Vol}}$	λ_{IF_X}	R^2
OLS	-0.0468	0.0267	0.631	0.504	58.3%
	(-1.11)	(0.06)	(2.20)	(3.26)	(52.3%)
	(-0.69)	(0.04)	(1.47)	(2.11)	
WLS	-0.0567	0.012	0.724	0.479	
	(-0.85)	(0.02)	(1.71)	(2.13)	

Since we are using end of year stock prices and returns in our analysis, we should only consider Q4-Q4 consumption growth. However, it is instructive to also consider the other correlations in table (XVII). They show that while Q2-Q2 and Q3-Q3 consumption growths are largely uncorrelated with IF_{-Vol} and IF_X, Q1-Q1 consumption growth is even more highly correlated with these innovations than Q4-Q4 consumption growth. Even more interestingly, IF_{-Vol} and IF_X do not drive out Q1-Q1 consumption growth in the cross-sectional regression. This fact is not surprising in itself as the innovation in consumption growth is a pricing factor in the long run risk model but is surprising in light of the fact that both Q4-Q4 and annual consumption growth are driven out by IF_{-Vol} and IF_X in the cross-sectional regression.

These results suggest that agents may plan a large part of their Q1 consumption at the end of the previous Q4 so that Q1-Q1 consumption growth is a better measure of the consumption planned by the agent when making her consumption-investment decision at the end of the previous Q4. This implies that Q1-Q1 consumption growth may be serving not only as a proxy for the innovations to the long run risk factor but also for the true consumption growth between the points of time at which the consumption-investment decision is made.

Another possibility is that Q4 and Q1 consumption has a larger proportion of goods and services that have a long run component. This is particularly plausible as Q4 and Q1 occur during the holiday season where consumption is more discretionary and, as Jagannathan, Marakani, Takehara, and Wang (2011) note, the composition of consumption during these quarters is qualitatively different from those of other quarters. This hypothesis can be tested using international data as the non-holiday affected Q2-Q2 consumption growth explains the cross section of stock returns better than the holiday affected Q4-Q4 consumption growth in the UK. However, the model will also have to be theoretically extended before this can be done as it currently treats all non-durables and services as being perfectly substitutable. Such a study is a subject for future research.

F. Relative Risk Aversion

Since it is largely F_X that predicts future dividend and consumption growth, we can, as pointed out, among others, by Hansen, Heaton, and Li (2008) and Kaltenbrunner and Lochstoer (2010), use the GMM estimate for the coefficient of scaled innovation of X (i.e. IF_X) in the stochastic discount factor, α_{IF_X}, to make an estimate of the preference for the early resolution of uncertainty, i.e. $\gamma - 1/\psi$. Since the value of the elasticity of intertemporal substitution (EIS), ψ, has to be large in long run risk models in order for them to be consistent with the low volatility of the real risk free rate, this value also provides an estimate of the relative risk aversion γ since $\gamma - 1/\psi \approx \gamma$ if $\psi > 1$.[34]

We obtain this estimate by first noting the relation (which follows from the identification $F_X \propto X$ and the fact that there is only one identified X component)

$$\begin{aligned} \alpha_{IF_X} &= \frac{\gamma - 1/\psi}{1 - \nu_1(1-\alpha\Delta t)} \left(\frac{\epsilon_t}{IF_{X,t}}\right) \\ &= \frac{\gamma - 1/\psi}{1 - \nu_1(1-\alpha\Delta t)} \left(\frac{X}{F_X}\right) \end{aligned} \tag{26}$$

where $IF_{X,t}$ and $\epsilon_t = \varphi_x \delta_x \sqrt{V}(Y_t - Y_{t-1})$ stand for the innovations of the second principal component and X respectively (note that $IF_{X,t}$ and ϵ_t are proportional to each other here so that their ratio is still independent of t). An estimate for $\frac{X}{F_X}$ can be obtained from the results of the regression in table (V). Using the relation (1), it is not difficult to see that coefficient obtained when regressing $\Delta c_{t+1} + \Delta c_{t+2}$ on X_t is given by $\sum_{i=0}^{1}(1-\alpha\Delta t)^i$. Hence, the coefficient obtained when regressing $\Delta c_{t+1} + \Delta c_{t+2}$ on F_X is

$$\sum_{i=0}^{1}(1-\alpha\Delta t)^i \frac{X}{F_X} \tag{27}$$

[34]We also note that a large value of ψ is not only strongly suggested by the low volatility of the real risk free rate but also by the analysis of household survey data by Vissing-Jørgensen and Attanasio (2003) and by the study of an elegant natural experiment by Kapoor and Ravi (2010).

By noting that the persistence of the X process is the same as that of F_X since they are proportional, we find α to be about 0.15 on the annual time scale. Using this value and the regression coefficient of 0.0123 obtained in table (V), we find that $\frac{F_X}{X} \approx 150$. This value, together with the annual estimate of $0.997^{12} \approx 0.97$ for ν_1 from Bansal and Yaron (2004) and the GMM estimates of 1.56 to 2.89 for the market price of risk of innovations to F_X, give an estimate of between 40 and 75 for $\gamma - 1/\psi$ (or equivalently γ since $\gamma \gg 1/\psi$). While high, this estimate is similar to the value of 60 obtained by Chen, Favilukis, and Ludvigson (2007).

The leverage of market dividend growth on long term consumption growth can be similarly estimated from the results in table (V). It is found to be about 3.3 when three year market dividend growth is used in the analysis. This value is remarkably similar to that proposed by Bansal and Yaron (2004).

It should be noted that the main reason that the risk aversion estimate here is much higher than that proposed by Bansal and Yaron (2004) is that the volatility of consumption growth after the structural break (1.85×10^{-4}) is much lower than that for the entire period for which data is available (4.92×10^{-4}). If we scale the relative risk aversion value estimates that we obtain by the ratio of these volatilities, we find that it is very similar to the value of 16 obtained by Bansal, Yaron, and Kiku (2007). Hence, it is possible that a long run risk model which accounts for structural breaks or regime shifts in the parameters will require a much lower relative risk aversion to explain asset prices as such a model can have a much higher unconditional volatility of consumption growth and still be consistent with the data. We also note that the standard errors for our estimate are large and we cannot rule that the relative risk aversion value is below 10 at the 1% level of significance.

VI. Conclusion

In this paper, we show that long run risk models in general, including those of Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009), imply that the log P/D ratios of financial assets have a strict factor structure when the intertemporal budget constraint of the marginal investor can be well approximated by the loglinear method of Campbell and Shiller (1988). Further, we demonstrate that these factors must be related to aggregate consumption growth and consumption growth volatility when there is a representative agent. When we restrict attention to the post-1942 data so as to account for the structural break documented by Marakani (2009), we find that the log P/D ratios of the 25 Fama-French portfolios have two significant factors, one of which is related to aggregate consumption growth volatility and the other to future dividend and real time aggregate consumption growth.

We also find these factors and their innovations do a reasonably good job of explaining the cross section of returns of not only the 25 portfolios from which they were formed but also three sets of ten portfolios based on long term reversal, short term reversal and the earnings to price ratio. The coefficients obtained from the cross sectional regressions are statistically and economically significant and have the right sign, and the zero beta rate is not significantly different from zero. Thus, we find that long run risk models of the type considered in the literature have the potential to explain financial market facts.

Our findings link the classical commonly used linear factor models in the finance literature with the more recent long run risk models. The crucial difference is that it is the factor structure of the component of returns orthogonal to contemporaneous dividend shocks which matters in long run risk models.

Beeler and Campbell (2009) point out that long run risk models imply counter-factually high predictability of long term aggregate consumption growth, long term dividend growth and future market volatility by the market price-dividend ratio. In this paper, we address the first two issues by showing that a log P/D factor does in fact predict long term dividend growth and real time consumption growth.

While we do not consider market volatility in this paper, we do find, in unreported results, some indicative evidence that the same log P/D factor also predicts market volatility. However, the predictability that we find is different from what is expected from the model since this factor does not predict consumption growth volatility. Beeler and Campbell (2009) also point out that long run risk models imply counter-intuitively high or infinite prices for real risk free consol bonds. This weakness of the long run risk model (and many other asset pricing models) is related to the fact that the variance of the permanent and transitory components of it's stochastic discount factor are inconsistent with the data as pointed out by Bakshi and Chabi-Yo (2011). These are issues to be addressed in future research.

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A. Log P/D Ratios in the General Long Run Risk Model

The methodology here closely follows that of Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007) as there are only two cases in the literature where solutions are available for models with Epstein-Zin preferences. The first case, which we are interested in here, is when the returns are loglinear in the state variables and the second is when $\psi = 1$.

Let c, $X_i, 1 \leq i \leq n$ and $V_j, 1 \leq j \leq m$ be the log consumption process, n processes that determine it's conditional growth rate and m processes that determine it's conditional growth rate volatility respectively. Let $d_l, l \leq 1 \leq L$ be the log dividend processes of L assets (in general, the lower case variables correspond to the logarithm of the upper case variables). We assume that these quantities follow the processes

$$
\begin{aligned}
c_{t+\Delta t} =& c_t + \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t + \sqrt{\sum_{j=1}^{m} \delta_{c,j}^2 V_{j,t}}\,(W_{t+\Delta t} - W_t) \\
& - \sum_{k=1}^{m} \varphi_{w,k}\sqrt{V_{k,t}}\,(Z_{k,t+\Delta t} - Z_{k,t})
\end{aligned} \tag{28}
$$

$$
X_{i,t+\Delta t} = X_{i,t}(1 - \alpha_i \Delta t) + \varphi_{i,x}\sqrt{\sum_{j=1}^{m} \delta_{x,i,j}^2 V_{j,t}}\,(Y_{i,t+\Delta t} - Y_{i,t}), 1 \leq i \leq n \tag{29}
$$

$$
V_{i,t+\Delta t} = V_{i,t} - \kappa_i(V_{i,t} - \bar{V}_i)\Delta t + \sigma_i\sqrt{V_{i,t}}\,(Z_{i,t+\Delta t} - Z_{i,t}), 1 \leq i \leq m \tag{30}
$$

$$
\begin{aligned}
d_{l,t+\Delta t} =& d_{l,t} + \left(\mu_l + \sum_{i=1}^{n} \phi_{l,i} X_{i,t}\right)\Delta t + \pi_{l,d}\left(\Delta c_{t+\Delta t} - \left(\mu + \sum_{i=1}^{n} X_{i,t}\right)\Delta t\right) \\
& + \sum_{i=1}^{n} \pi_{i,l,x}(X_{i,t+\Delta t} - X_{i,t}(1 - \alpha_i \Delta t)) \\
& + \sum_{j=1}^{m} \pi_{j,l,w}\sigma_j\sqrt{V_{j,t}}\,(Z_{j,t+\Delta t} - Z_{j,t}) \\
& + \sqrt{\sum_{k=1}^{m} \delta_{l,d,k}^2 V_{k,t}}\,(B_{t+\Delta t} - B_t)
\end{aligned} \tag{31}
$$

where W, $Y_i, 1 \leq i \leq n$, $Z_j, 1 \leq j \leq m$ and B are independent Brownian processes and $\sum_{i=1}^{m} \delta_{c,i}^2 = \sum_{j=1}^{m} \delta_{x,i,j}^2 = \sum_{k=1}^{m} \delta_{l,d,k}^2 = 1$. We have written the equations in this form (with the time step being Δt rather than 1) to make the time scale dependence of the parameters explicit so that the connection with the continuous time solution can be made in a straightforward manner. We also define the consumption and dividend variables as rates since they are flow variables. This means, for example, that consumption from time t to $t+\Delta t$ is given by $C_{t+\Delta t}\Delta t$.

Since the consumer preferences are of the Epstein-Zin type

$$U_t = ((1-\delta)(C_t\Delta t)^{\frac{1-\gamma}{\theta}} + \delta E_t[U_{t+\Delta t}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \tag{32}$$

where

$$\theta = \frac{1-\gamma}{1-1/\psi} \tag{33}$$

the log stochastic discount factor in discrete time can be written as

$$m_{t+\Delta t} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+\Delta t} + (\theta - 1) r_{c,t+\Delta t} \tag{34}$$

where $r_{c,t+\Delta t}$ is the continuously compounded rate of return on the wealth W (which is the asset that delivers a dividend of per capita consumption at every time period) from t to $t+\Delta t$. Since we assume complete markets,

$$E_t[\exp(m_{t+\Delta t} + r_{c,t+\Delta t})] = 1 \tag{35}$$

must hold.

The loglinear approximation pioneered by Campbell and Shiller (1988) allows us to write

$$r_{c,t+\Delta t} = \nu_0 + \nu_1(w_{t+\Delta t} - c_{t+\Delta t}) - (w_t - c_t) + \Delta c_{t+\Delta t} \tag{36}$$

where

$$\nu_0 = \log(\Delta t + \exp(\overline{w-c})) - \nu_1(\overline{w-c}) \approx \exp(\overline{c-w})(1 + (\overline{c-w}))\Delta t \tag{37}$$

$$\nu_1 = \frac{1}{1 + \exp(\overline{c-w})\Delta t} \approx 1 - \exp(\overline{c-w})\Delta t \tag{38}$$

(the approximation holds when Δt is small) where the bar stands for the mean value. We further assume that the log wealth to consumption ratio can be written as

$$w_t - c_t = A_0 + \sum_{i=1}^{n} A_{1,i} X_{i,t} + \sum_{j=1}^{m} A_{2,j} V_{j,t} \tag{39}$$

and justify this below. (This approach is standard and followed by Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007) and Zhou and Zhu (2009) as the only non-trivial models with Epstein-Zin preferences which can be solved are those where the consumption to wealth ratio is loglinear in the state variables as above or where $\psi = 1$, as in the model of Hansen, Heaton, and Li (2008)).

Substituting (34), (36) and (39) into (35), using the fact that

$$\log E_t[\exp A(W_{t+\Delta t} - W_t)] = \frac{A^2 \Delta t}{2} \tag{40}$$

for any $A \in \mathbb{R}$ and Wiener process W, and that (35) should hold for any possible attainable combination of state variables (X_i, V_j), we obtain a set of equations which enable us to solve for A_0, $A_{1,i}, 1 \leq i \leq n$ and $A_{2,j}, 1 \leq j \leq m$. The fact that such a set of equations with non-vacuous solutions exist justifies the assumption (39).

The set of equations for $A_{1,i}$ are

$$(1-\gamma)\Delta t + \theta A_{1,i}(\nu_1(1 - \alpha_i \Delta t) - 1) = 0 \tag{41}$$

so that

$$A_{1,i} = \frac{(1 - \frac{1}{\psi})\Delta t}{1 - \nu_1(1 - \alpha_i \Delta t)} \tag{42}$$

which, in the limit $\Delta t \to 0$, becomes $A_{1,i} = \frac{1-1/\psi}{\exp(\overline{c-w})+\alpha_i}$. This is the same result as that obtained by Zhou and Zhu (2009), where there is only one X variable, once we relate his notation of g_1 for $\exp(\overline{c-w})$ and allow for the negative sign which arises from his definition of A_1 in terms of the consumption to wealth ratio. Once we set $\Delta t = 1$ and relabel ν_1 as κ_1 and α_i as $1-\rho$ (again, there being only one X state variable) to match the notation of Bansal and Yaron (2004), we find that our result also matches their's.

The analogous set of equations which enables us to solve for $A_{2,j}, 1 \leq j \leq m$ is

$$\begin{aligned}&\frac{(1-\gamma)^2\delta_{c,j}^2\Delta t}{2} + \theta A_{2,j}(\nu_1(1-\kappa_j\Delta t)-1)\\&+\frac{\Delta t}{2}\left(\left(\theta\nu_1\sum_{i=1}^{n}A_{1,i}\varphi_{x,i}\delta_{x,i,j}\right)^2 + (\theta\nu_1 A_{2,j}\sigma_j - (1-\gamma)\varphi_{w,j})^2\right) = 0\end{aligned} \tag{43}$$

Since these equations are quadratic, there are two solutions for each $A_{2,j}$. However, one of them diverges when $\sigma_j \to 0$. Hence, the other solution is the one which is relevant to the model. The final equation, which allows us to solve for A_0, is

$$\theta\left(\log\delta + \nu_0 + (\nu_1 - 1)A_0 + \nu_1\sum_{j=1}^{m}A_{2,j}\kappa_j\Delta t\bar{V}_j\right) + (1-\gamma)\mu\Delta t = 0 \tag{44}$$

Putting the values for A_0, $A_{1,i}, 1 \leq i \leq n$ and $A_{2,j}, 1 \leq j \leq m$ into (39) and using (36) and (34), we obtain the log stochastic discount factor

$$\begin{aligned}m_{t+\Delta t} =& \Delta t\left(\Gamma_0 + \sum_{i=1}^{n}\Gamma_{1,i}X_{i,t} + \sum_{j=1}^{m}\Gamma_{2,j}V_{j,t}\right)\\&-\alpha_c\sqrt{\sum_{j=1}^{m}\delta_{c,j}^2 V_{j,t}}(W_{t+\Delta t}-W_t)\\&-\sum_{i=1}^{n}\alpha_{x,i}\sqrt{\sum_{j=1}^{m}\delta_{x,j}^2 V_{j,t}}(Y_{i,t+\Delta t}-Y_{i,t})\\&-\sum_{j=1}^{m}\alpha_{v,j}\sqrt{V_{j,t}}(Z_{j,t+\Delta t}-Z_{j,t})\end{aligned} \tag{45}$$

where $\Gamma_{1,i} = 1/\psi$, $\alpha_c = \gamma$ and $\alpha_{x,i} = \frac{\gamma - 1/\psi}{1-\nu_1(1-\alpha_i \Delta t)}$. The expression for $\alpha_{v,j}$ is complicated and does not directly concern us here.

Using the process for dividend growth (31), we can use a similar loglinear approximation to write the return for asset l as

$$r_{l,t+\Delta t} = \nu_{0,l} + \nu_{1,l}(p_{l,t+\Delta t} - d_{l,t+\Delta t}) - (p_{l,t} - d_{l,t}) + \Delta d_{l,t+\Delta t} \tag{46}$$

where

$$\begin{aligned} \nu_{0,l} &= \log(\Delta t + \exp(\overline{d_l - p_l})) - \nu_{1,l}(\overline{p_l - d_l}) \\ &\approx \exp(\overline{d_l - p_l})(1 + \overline{d_l - p_l})\Delta t \end{aligned} \tag{47}$$

$$\nu_{1,l} = \frac{1}{1 + \exp(\overline{d_l - p_l})\Delta t} \approx 1 - \exp(\overline{d_l - p_l})\Delta t \tag{48}$$

As before, we assume that $\log\left(\frac{P_t}{D_t}\right)$ can be written as

$$\log\left(\frac{P_{l,t}}{D_{l,t}}\right) = p_{l,t} - d_{l,t} = A_{0,l} + \sum_{i=1}^{n} A_{1,l,i} X_{i,t} + \sum_{j=1}^{m} A_{2,l,j} V_{j,t} \tag{49}$$

We put (49) into (46) and use the fact that (35) must hold for any possible attainable combination of state variables (X_i, V_j) to obtain a set of equations which enables us to solve for $A_{0,l}$, $A_{1,l,i}, 1 \leq i \leq n$ and $A_{2,l,j}, 1 \leq j \leq m$. The fact that such a set of equations with non-vacuous solutions exist justifies the assumption (49).

The equations for $A_{1,l,i}, 1 \leq i \leq n, 1 \leq l \leq L$ are

$$(\phi_{l,i} - 1/\psi)\Delta t - A_{1,l,i}(1 - \nu_{1,l}(1 - \alpha_i \Delta t)) = 0 \tag{50}$$

which give

$$A_{1,l,i} = \frac{(\phi_{l,i} - 1/\psi)\Delta t}{1 - \nu_{1,l}(1 - \alpha_i \Delta t)} \tag{51}$$

As with the solution for $A_{1,i}, 1 \leq i \leq n$, this solution agrees with the continuous time one (with $n = 1, m = 2$) of Zhou and Zhu (2009) and the discrete time one (with $n = m = 1$) of Bansal and Yaron (2004) and (Bansal, Yaron, and Kiku 2007).

The equations for $A_{2,l,j}, 1 \leq j \leq m, 1 \leq l \leq L$ are quadratic in nature and fairly complex (as for $A_{2,j}$, the solutions which do not diverge as $\sigma_j \to 0$ are chosen). Since their precise structure does not concern us here, we do not include them for brevity. Similarly, we do not include the equation for $A_{0,l}, 1 \leq l \leq L$.[35]

It must be noted that, as the equations for $A_{2,j}, 1 \leq j \leq m$ and $A_{2,l,j}, 1 \leq j \leq m, 1 \leq l \leq L$ are quadratic in nature, real solutions are not guaranteed. Our numerical experiments indicate that this is not a serious concern as several sets of reasonable parameter values do not give rise to this problem (this is also shown by Zhou and Zhu (2009)). If this is a concern, we can replace the volatility processes by Ornstein-Uhlenbeck ones as done by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007). However, such volatility processes suffer from the problem of admitting negative values even in continuous time. This can be quite serious, even for some common parameter values, as pointed out by Beeler and Campbell (2009). The square root processes used here can also give rise to negative values in discrete time but the probability of this occurring for reasonable parameter values is minuscule and our numerical experiments confirm this. Since both ways of modeling volatility have issues but have received wide attention in the literature and there is no known alternative for which analytical solutions can be derived, we use results which hold for both of them.

[35] They are available upon request from the authors.

B. Testing Long Run Risk Models : Monte Carlo Evidence

A. The Model

For the purpose of analyzing the performance of the asset pricing tests, we use the long run risk model of Bansal and Yaron (2004). In this model, the per capita consumption and dividend growth rates Δc and Δd (for M assets indexed by l) and their common persistent component x are assumed to follow the processes (see Bansal and Yaron (2004))

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \tag{52}$$

$$x_{t+1} = \rho x_t + \varphi_x \sigma_t e_{t+1} \tag{53}$$

$$\Delta d_{l,t+1} = \mu_{l,d} + \phi_l x_t + \varphi_{l,d} \sigma u_{l,t+1}, 1 \leq l \leq M \tag{54}$$

$$\sigma^2_{t+1} = \sigma^2 + \nu(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \tag{55}$$

where the shocks e_{t+1}, η_{t+1} and w_{t+1} are taken to be independent standard normals for parsimony. $u_{l,t+1}$ is a vector of normally distributed shocks with covariance V_u which is independent of e, η and w. In the simulations, V_u is set so as to fit the factor structure of returns. (Note that we follow the convention that lowercase characters stand for the logarithm of quantities denoted by the corresponding uppercase characters.)

Consumers in the model have Epstein-Zin preferences (as defined by Epstein and Zin (1989))

$$U_t = ((1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}})^{\frac{\theta}{1-\gamma}} \tag{56}$$

with $\gamma > 1/\psi$. This implies that they prefer early resolution of uncertainty and that persistent consumption and volatility shocks have a positive market price of risk. With these preferences, asset returns satisfy

$$E_t \left[\delta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta/\psi} R_{a,t+1}^{-(1-\theta)} R_{i,t+1} \right] = 1 \tag{57}$$

where C is per capita consumption, R_a is the gross return on an asset that pays a dividend of per capita consumption, R_i is the asset return, $0 < \delta < 1$ is the time discount factor, γ is the relative risk aversion, ψ is the intertemporal elasticity of substitution (IES) and θ is defined to be

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}} \tag{58}$$

The log P/D ratios of assets in this economy have a factor structure (within the loglinear approximation) with the factors being x_t and σ_t^2. In other words, if $z_{i,t}$ is the log P/D ratio of asset i, we have

$$z_{i,t} = A_{0,i} + A_{1,i}x_t + A_{2,i}\sigma_t^2 \tag{59}$$

This is shown for this particular model by Bansal and Yaron (2004) and similar results for related models are shown by Bansal, Yaron, and Kiku (2007), Drechsler and Yaron (2011), Zhou and Zhu (2009), Ferson, Nallareddy, and Xie (2009) and in appendix A of this paper. Since the dividend processes of the assets are specified in this model, the relation above gives the time series of their prices for a given realization of the random variables. Hence, the prices and other quantities of interest in this economy are readily simulated.

B. Monte Carlo Simulation of the Model

We use the global and asset specific parameters summarized in tables (XIX) and (XX) for the simulations below. We first note that these parameters generate economic moments (calculated from 500 simulations of the long run risk economy) which are roughly in line with the values observed in post-1942 (to account for the structural break identified by Marakani (2009)) US consumption and return data as shown in table (XXI). When realistic noise is added to the log P/D ratios as described below, they are also compatible with the predictability of real time consumption growth in the data as seen from the numbers in table (XXII). One moment which does not match well is the standard deviation of the real risk free rate which is *much* smaller in the simulations than in the data. This, however, as argued by Beeler and Campbell (2009), points to a strength rather than a weakness of the long run risk model as most models struggle to make this quantity low enough. Further, as we argue in the next section, this quantity is very noisily measured which means that the reported standard deviation would be significantly larger than the actual one.[36]

Table XIX
Global parameters for the simulation

Global parameters for the simulation (the time unit is one year). μ represents the unconditional mean of consumption growth, σ it's conditional volatility, ρ the first order autocorrelation of the long run risk state variable x, φ_x the conditional volatility of x in relation to that of consumption growth, ν the first order autocorrelation of volatility, σ_w the volatility of volatility, γ the relative risk aversion, ψ the elasticity of intertemporal substitution and δ the time preference.

Parameter	Value
μ	0.02
σ	0.012
ρ	0.85
φ_x	0.45
ν	0.99
σ_w	10^{-5}
γ	25
ψ	1.5
δ	0.994

The scaled eigenvalues of the covariance matrix of the post-1942 continuously compounded excess returns of the 25 Fama-French portfolios formed on the basis of size and book to market ratio are tabulated in table (XXIII) together with the mean, 5th and 95th percentiles of the corresponding values obtained in 500 simulations of the economy for the same time period (65 years).[37] Since the first few eigenvalues, which are of principal interest, are very similar to those in the data, the model replicates the observed factor structure of excess returns quite well.

The model also replicates the observed factor structure of log P/D ratios fairly well. This is best seen from the normalized eigenvalues for the covariance matrix of the log P/D ratios

[36] Measurement error (in either inflation or dividends) can also account for the somewhat low standard deviation of real dividend growth of the portfolios in the simulations.

[37] The model was actually simulated for 165 years with the data for the first 100 years being discarded so as to minimize the effect of the assumed initial values of the dynamic quantities.

of the assets, both from the data as well as the simulations, which are tabulated in table (XXIV). The model's two factor structure is highly evident here as all the eigenvalues after the second one are zero. To better reflect the data and investigate the possible consequences of the inclusion of small, irrelevant factors into the long run risk model, we added white noise with a variance of 20% of the simulated values to the log P/D ratios. The introduction of this noise can also be thought of as representing measurement error in the prices or dividends brought about due to liquidity issues or other market imperfections. The normalized eigenvalues after adding this noise are summarized in table (XXV). From it, we see that the model is able to replicate the key elements of this factor structure after adding the noise.[38]

We thus see that the long run risk model being simulated here is compatible not only with many of the important observed moments of macroeconomic quantities but also with the observed factor structure of excess returns and P/D ratios. Given this, it is interesting to examine the performance of different asset pricing tests for long run risk models within the context of these simulations. This will enable the study of the effect of finite sample size and measurement noise on the efficacy of these tests and will point to the choice of test to be used in this paper. Since we are particularly interested in examining the impact of measurement noise on these tests, we first turn to the task of establishing a reasonable estimate for the size of this noise for two important quantities in long run risk models, the consumption growth and the real risk free rate.

[38]Note that it is not necessary to replicate the features of the small factors as these represent a very small fraction of the variance and are not economically interesting.

Table XX
Asset-specific parameters for the simulation.

Asset-specific parameters for the simulation. The assets are indexed by l. $\mu_{l,d}$ represents the unconditional mean of the dividend growth for asset l, ϕ_l the dependence of predictable dividend growth on the long run risk state variable x and $\varphi_{l,d}$ the idiosyncratic volatility of dividend growth.

Parameters for the asset dividend growths

l	$\mu_{l,d}$	ϕ_l	$\varphi_{l,d}$
1	−0.0286	1.7834	19.1677
2	0.0889	3.7689	21.7081
3	0.0160	3.2545	19.4655
4	0.0456	3.4405	23.5766
5	0.0471	2.6758	24.0000
6	0.0907	4.6342	16.6065
7	0.0778	5.8088	16.3543
8	0.0457	2.4918	8.5237
9	0.0928	9.5089	24.0000
10	−0.0145	5.5979	24.0000
11	−0.0012	4.8912	24.0000
12	0.0821	8.5459	22.0032
13	0.0556	10.9271	8.9635
14	0.0272	6.0810	21.8607
15	0.0926	5.1230	24.0000
16	0.0454	5.1540	6.0000
17	0.0327	3.0965	21.1709
18	0.0317	3.3548	16.4485
19	0.0147	3.5232	23.0091
20	0.0619	3.3028	6.6980
21	0.0167	2.5690	12.5081
22	0.0421	10.8271	6.0000
23	0.0901	3.7845	11.6097
24	0.0436	2.5953	24.0000
25	0.0788	3.7323	11.0877

Table XXI
Model implied moments for important economic quantities compared with the data

The model implied moments are obtained from 500 simulations.

Moment	Data	Simulation mean	5th percentile	95th percentile
$E[\Delta c_t]$	0.0199	0.0200	0.0153	0.0246
$\text{Std}[\Delta c_t]$	0.0136	0.0151	0.0105	0.0194
$AC(1)[\Delta c_t]$	0.243	0.320	0.148	0.488
$E[r_{f,t}]$	0.0059	0.0035	-0.0012	0.0079
$\text{Std}[r_{f,t}]$	0.0343	0.0067	0.0045	0.0089
$\text{Min}[r_{l,t} - r_{f,t}]$	0.010	0.018	-0.012	0.049
$\text{Max}[r_{l,t} - r_{f,t}]$	0.133	0.209	0.131	0.292
$\text{Min}\, E[\Delta d_{l,t}]$	-0.023	-0.030	-0.062	0.002
$\text{Max}\, E[\Delta d_{l,t}]$	0.105	0.104	0.070	0.149
$\text{Min}\, \text{Std}[\Delta d_{l,t}]$	0.087	0.085	0.075	0.095
$\text{Max}\, \text{Std}[\Delta d_{l,t}]$	0.385	0.306	0.279	0.333

Table XXII
Predictability of consumption growth in the model and in the data.

For the data, we use real time consumption growth as the measure of consumption growth. The results for the model are derived from 1000 simulations over 165 years with the data for the first 100 years being dropped so as to limit the impact of initial values on the numbers.

Data	Simulation mean	5th percentile	95th percentile
17.4%	32.6%	10.6%	55.2%

Table XXIII
Factor structure of excess returns in the model and in the data

Eigenvalues of the covariance matrix of the continuously compounded excess returns of the 25 Fama-French portfolios as well as those obtained by simulating the model.

Eigenvalues of the covariance matrix of excess returns

Data	Simulation mean	5th percentile	95th percentile
1.00000	1.00000	1.00000	1.00000
0.06052	0.06171	0.04705	0.07889
0.04741	0.03926	0.02994	0.04970
0.01280	0.01135	0.00871	0.01403
0.00823	0.00807	0.00637	0.01010
0.00626	0.00667	0.00531	0.00824
0.00535	0.00573	0.00455	0.00711
0.00389	0.00497	0.00399	0.00613
0.00339	0.00433	0.00351	0.00541
0.00316	0.00375	0.00302	0.00460
0.00288	0.00331	0.00267	0.00403
0.00231	0.00294	0.00240	0.00359
0.00207	0.00263	0.00214	0.00324
0.00200	0.00236	0.00191	0.00289
0.00149	0.00213	0.00170	0.00265
0.00142	0.00191	0.00152	0.00234
0.00132	0.00171	0.00136	0.00212
0.00108	0.00151	0.00118	0.00186
0.00099	0.00132	0.00106	0.00164
0.00097	0.00112	0.00087	0.00139
0.00074	0.00076	0.00059	0.00095
0.00067	0.00058	0.00045	0.00075
0.00056	0.00040	0.00030	0.00050
0.00045	0.00030	0.00023	0.00038
0.00043	0.00023	0.00017	0.00029

Table XXIV
Factor structure of log P/D ratios in the model and in the data.

Eigenvalues of the covariance matrix of the log P/D ratios of the 25 Fama-French portfolios as well as those obtained by simulating the model.

Eigenvalues of the covariance matrix of log P/D ratios

Data	Simulation mean	5th percentile	95th percentile
1.00000	1.00000	1.00000	1.00000
0.06041	0.03598	0.01272	0.07067
0.01669	0.00000	0.00000	0.00000
0.01169	0.00000	0.00000	0.00000
0.00627	0.00000	0.00000	0.00000
0.00522	0.00000	0.00000	0.00000
0.00494	0.00000	0.00000	0.00000
0.00318	0.00000	0.00000	0.00000
0.00245	0.00000	0.00000	0.00000
0.00238	0.00000	0.00000	0.00000
0.00215	0.00000	0.00000	0.00000
0.00168	0.00000	0.00000	0.00000
0.00137	0.00000	0.00000	0.00000
0.00101	0.00000	0.00000	0.00000
0.00094	0.00000	0.00000	0.00000
0.00085	0.00000	0.00000	0.00000
0.00072	0.00000	0.00000	0.00000
0.00063	0.00000	0.00000	0.00000
0.00052	0.00000	0.00000	0.00000
0.00049	0.00000	0.00000	0.00000
0.00046	0.00000	0.00000	0.00000
0.00040	0.00000	0.00000	0.00000
0.00028	0.00000	0.00000	0.00000
0.00022	0.00000	0.00000	0.00000
0.00018	0.00000	0.00000	0.00000

Table XXV
Factor structure of log P/D ratios in the model with noise and in the data.

Eigenvalues of the covariance matrix of the log P/D ratios of the 25 Fama-French portfolios as well as those obtained by simulating the model and adding some noise to the result.

Eigenvalues of the covariance matrix of noisy log P/D ratios

Data	Simulation mean	5th percentile	95th percentile
1.00000	1.00000	1.00000	1.00000
0.06041	0.04536	0.02144	0.08128
0.01669	0.01451	0.01323	0.01577
0.01169	0.01337	0.01234	0.01442
0.00627	0.01251	0.01160	0.01352
0.00522	0.01179	0.01095	0.01269
0.00494	0.01114	0.01046	0.01183
0.00318	0.01057	0.00989	0.01132
0.00245	0.01003	0.00936	0.01071
0.00238	0.00952	0.00877	0.01022
0.00215	0.00904	0.00842	0.00972
0.00168	0.00859	0.00800	0.00918
0.00137	0.00813	0.00757	0.00872
0.00101	0.00771	0.00712	0.00837
0.00094	0.00730	0.00677	0.00786
0.00085	0.00692	0.00639	0.00751
0.00072	0.00652	0.00603	0.00706
0.00063	0.00615	0.00567	0.00664
0.00052	0.00579	0.00533	0.00627
0.00049	0.00541	0.00496	0.00588
0.00046	0.00506	0.00464	0.00554
0.00040	0.00470	0.00429	0.00514
0.00028	0.00433	0.00388	0.00477
0.00022	0.00394	0.00350	0.00437
0.00018	0.00345	0.00295	0.00390

C. Measurement Error

We do so by analyzing the degree of correlation between different measures for the same fundamental macroeconomic quantities. For consumption growth, we use the estimates of consumption growth derived from the continuously revised NIPA tables as well as those from the real time database maintained by the Federal Reserve Bank of St. Louis (described in detail by Croushore and Stark (2001)). Regressing these estimates against each other leads to the results in table (XXVI). The R^2 of 67% or about $\frac{2}{3}$ indicates that the variance of measurement noise in consumption growth is about half of the variance of actual consumption growth. We thus simulate measured consumption growth as actual consumption growth plus iid noise with half it's realized variance in that simulation.

Table XXVI
Measurment error in consumption growth

Regression of the conventional revised measure of consumption growth Δc on the corresponding real time measure Δc^{RT}.

	Intercept	Δc^{RT}	R^2
Δc	0.0060 (0.0019)	0.838 (0.092)	67.0%

Similarly, we regress three measures of the real risk free rate on each other to estimate the amount of measurement noise in it. We use the three measures considered by Marakani (2009), i.e. estimates constructed with the use of lagged, realized and expected inflation. From the results tabulated in table (XXVII), we see that the R^2 of each of the regressions is quite low with the average being under 33%. This indicates that the measurement noise in the reported real risk free rate has about twice the variance of the underlying quantity. Hence, for the simulations, we model the measured real risk free rate as the actual risk free rate plus iid noise with twice it's realized variance.

Table XXVII
Measurement error in the real risk free rate

Regression of three measures of the real risk free rate on each other. The three measures are computed using the lagged, realized and expected inflation. The regressions are restricted to the post-1946 period as expected inflation data is only available for it.

Regression of $r_{f,t}^{\text{lagged}}$ against $r_{f,t}^{\text{realized}}$

Coefficient	Estimate (Std. Err.)
Intercept	0.0046 (0.0028)
$r_{f,t}^{\text{realized}}$	0.454 (0.106)
R^2	23.6%

Regression of $r_{f,t}^{\text{lagged}}$ against $r_{f,t}^{\text{expected}}$

Coefficient	Estimate (Std. Err.)
Intercept	-0.0023 (0.0030)
$r_{f,t}^{\text{expected}}$	0.890 (0.145)
R^2	38.6%

Regression of $r_{f,t}^{\text{realized}}$ against $r_{f,t}^{\text{expected}}$

Coefficient	Estimate (Std. Err.)
Intercept	-0.0007 (0.0035)
$r_{f,t}^{\text{expected}}$	0.859 (0.169)
R^2	30.4%

D. Type I error of Asset Pricing Tests with Respect to the Long Run Risk Model

We now analyze the performance of tests of four different asset pricing restrictions of the long run risk model in order to determine which is the most reasonable one to use in the analysis in this paper. The first two asset pricing restrictions that we consider are related to the one analyzed by Ferson, Nallareddy, and Xie (2009).[39] Of these, the first is[40]

$$
\begin{aligned}
E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\mathrm{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx &\beta_{\tilde{x}}\lambda_{\tilde{x}} + \beta_{\tilde{\sigma^2}}\lambda_{\tilde{\sigma^2}} + \sum_{i=1}^{2}\beta_{\tilde{\epsilon}}\lambda_{\tilde{\epsilon}} \\
&+ \sum_{j=1}^{2}\beta_{\tilde{w}}\lambda_{\tilde{w}}
\end{aligned} \tag{60}
$$

where the returns $r_{i,t}$ are continuously compounded, $\tilde{x}$ and $\tilde{\sigma^2}$ are the estimated values of x_t and σ_t^2 (note from the subscript that these are *lagged* values), and $\tilde{\epsilon}$ and $\tilde{w}$ are the estimated values of the innovations of these processes. x and σ^2 are estimated in the same manner as by Bansal, Yaron, and Kiku (2007) and Ferson, Nallareddy, and Xie (2009), i.e. by the use of the following regressions

$$
\Delta c_{t+\Delta t} = \alpha_0 + \alpha_1 z_{m,t} + \alpha_2 r_{f,t} + \sigma_t \eta_{t+\Delta t}\sqrt{\Delta t} \tag{61}
$$

$$
\tilde{x}_t = \alpha_0 - \mu + \alpha_1 z_{m,t} + \alpha_2 r_{f,t} \tag{62}
$$

$$
\tilde{x}_{t+\Delta t} = \rho\tilde{x}_t + \tilde{\epsilon}_{t+\Delta t}\sqrt{\Delta t} \tag{63}
$$

$$
\sigma_t^2\eta_{t+\Delta t}^2\Delta t = \beta_0 + \beta_1 z_{m,t} + \beta_2 r_{f,t} + \omega_{t+\Delta t} \tag{64}
$$

$$
\tilde{\sigma}_t^2\Delta t = \beta_0 + \beta_1 z_{m,t} + \beta_2 r_{f,t} \tag{65}
$$

$$
\tilde{\sigma}_{t+\Delta t}^2 = \nu\tilde{\sigma}_t^2 + \tilde{w}_{t+\Delta t}\sqrt{\Delta t} \tag{66}
$$

where $z_{m,t}$ is the log market P/D ratio (taken to be the log P/D ratio of the first asset in the simulations) and Δt is one year. The second asset pricing restriction that we consider comes from considering only the innovations to the stochastic discount factor as in Ferson, Nallareddy, and Xie (2009). This simplifies (60) to

$$
E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\mathrm{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \sum_{i=1}^{2}\beta_{\tilde{\epsilon}}\lambda_{\tilde{\epsilon}} + \sum_{j=1}^{2}\beta_{\tilde{w}}\lambda_{\tilde{w}} \tag{67}
$$

[39]Ferson, Nallareddy, and Xie (2009) use GMM with the Euler moment restrictions in the SDF framework. We use the beta representation which is approximate but quite accurate when dealing with continuously compounded returns.

[40]Note that we don't need a $\beta_{\Delta c}$ term as there is no contemporaneous correlation between the dividend growth and consumption growth innovations

The third and fourth asset pricing restrictions that we consider are analogous but use the two largest estimated log P/D ratio factors instead of the log market P/D ratio and the real risk free rate as they should also span x and σ^2. The principal idea behind this approach is that given the null, they should be more accurately estimated in the presence of measurement error since they are estimated using multiple assets. The asset pricing restriction analogous to (60) is then given by

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \sum_{i=1}^{2} \beta_{F_i}\lambda_{F_i} + \sum_{j=1}^{2} \beta_{IF_j}\lambda_{IF_j} \tag{68}$$

where F_i and IF_i are the ith principal components of the log P/D ratios of the assets and their estimated innovations respectively (the latter are estimated by fitting the former to an AR(1) process). The asset pricing restriction analogous to (67), which only uses the estimated innovations, is then

$$E[r_{i,t+\Delta t} - r_{f,t}] + \frac{1}{2}\text{Var}[r_{i,t+\Delta t} - r_{f,t}] \approx \sum_{j=1}^{2} \beta_{IF_j}\lambda_{IF_j} \tag{69}$$

We examine whether the hypothesis that the factors being considered are useless is rejected by the cross sectional regression methodology. This is done using the Wald test for the risk premia of the factors with their covariance matrix being estimated in the standard manner (see for eg., Shanken (1992) and Shanken and Zhou (2007)). The rejection frequencies for each of these tests in 1000 simulations are reported in table (XXVIII). The results show that the test of the asset pricing restriction involving the log P/D ratio factors (which also include noise calibrated to fit the observed factor structure of log P/D ratios) and/or their innovations display much greater power than those involving the estimated long run risk processes and their innovations. Hence, we use the former in our analysis in this paper.

Table XXVIII
Power of the two type of tests tested in the simulation

Rejection frequencies for the hypothesis that the λs of the relevant factors are zero.

Hypothesis	Non-rejection rate		
	p=0.10	p=0.05	p=0.01
$\lambda_{\tilde{x}}, \lambda_{\tilde{\sigma^2}}, \lambda_{\tilde{\epsilon}}, \lambda_{\tilde{w}} = 0$	14.9%	26.4%	48.8%
$\lambda_{\tilde{\epsilon}}, \lambda_{\tilde{w}} = 0$	12.3%	24.0%	50.4%
$\lambda_{F_1}, \lambda_{F_2}, \lambda_{IF_1}, \lambda_{IF_2} = 0$	0	0.2%	0.4%
$\lambda_{IF_1}, \lambda_{IF_2} = 0$	0.4%	0.6%	1.5%

E. Conclusion

In this appendix, we simulate a 25 asset long run risk economy with parameters chosen so as to match key economic and financial moments with those in U.S. economic and financial data. We analyze the type I error of different asset pricing tests within this economy and find, when realistic measurement noise is introduced into it, that tests using estimates of the long run risk components derived from projections of consumption growth onto the log market price dividend ratio and real risk free rate display high type I error while those estimating the same components using the principal components of the log price dividend ratios of the assets do not do so. This implies that the latter type of tests have a more desirable profile. Hence, we use such tests in this paper.

C. Out of Sample Tests

Table XXIX
Out of sample test for the relation between the first two principal components and consumption growth volatility

Results of regressing real annual market dividend growth against lagged $F_1^{a,os}$ and $F_2^{a,os}$, the out of sample estimates of the first and second log P/D factors. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure of Newey and West (1994).

Regression of 24 quarter consumption growth volatility on $F_1^{a,os}$ and $F_2^{a,os}$

	Intercept	$F_1^{a,os}$	$F_2^{a,os}$	R^2
v_t^{24}	0.171^{***} (0.016)	-0.0050^{***} (0.0007)	0.0022 (0.0026)	74.1%

To check the robustness of the results, we estimated the rotation matrices relating the log price dividend ratios of the portfolios to their first two principal components only using data from 1943 to 1975 and used them to construct out of sample factors from 1975 to 2008. We found that these estimated out of sample factors also track consumption growth volatility and predict market dividend and real time consumption growth in a manner similar to that documented for the in sample factors.

The results of regressing 24 quarter consumption growth volatility on the estimated out of sample factors, summarized in table (XXIX), show that the relation found in the paper is robust. Specifically, consumption growth volatility is found to be very significantly negatively related to the first out of sample factor $F_1^{a,os}$ and to be unrelated to the second out of sample factor $F_2^{a,os}$.

The predictability of real time consumption and market dividend growth using the out of sample factors are summarized in table (XXX). As can be seen, only the second factor is relevant in predicting real time consumption growth and market dividend growth. The result for the three year market dividend growth seems marginal but that is because the number of data points is much smaller and the R^2 of the regression is still found to be quite high.

Table XXX
Out of sample test for the relation between the first two principal components and future market dividend and real time consumption growth

Results of regressing real annual market dividend growth and real time consumption growth (Δc^{RT}) against lagged $F_1^{a,os}$ and $F_2^{a,os}$, the out of sample estimates of the first and second log P/D factors. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure of Newey and West (1994). The regressions using the log market price dividend ratio use data from 1976 onwards in order to be consistent with the others.

Regression of market dividend growth on $F_1^{a,os}$ and $F_2^{a,os}$ and the log market price dividend ratio

	$F_1^{a,os}$	$F_2^{a,os}$	$\log(P/D)_m$	R^2
1 yr. Market div. growth	-0.0066 (0.0055)	0.0491*** (0.0183)		20.8%
			0.012 (0.036)	0.8%
3 yr. Market div. growth	0.0026 (0.0263)	0.0593 (0.0453)		13.4%
			0.066 (0.138)	5.7%

Regression of real time annual consumption growth on lagged values of $F_1^{a,os}$ and $F_2^{a,os}$.

	$F_1^{a,os}$	$F_2^{a,os}$	R^2
Δc_{t+1}^{RT}	$4.1 \times 10^{-4}(0.0010)$	0.0063** (0.0031)	13.9%
Δc_{t+2}^{RT}	$5.5 \times 10^{-4}(6.8 \times 10^{-4})$	0.0045** (0.0016)	5.8%
$\Delta c_{t+1}^{RT} + \Delta c_{t+2}^{RT}$	$7.6 \times 10^{-4}(1.5 \times 10^{-3})$	0.0123** (0.0050)	18.9%

D. Robust Test Statistics

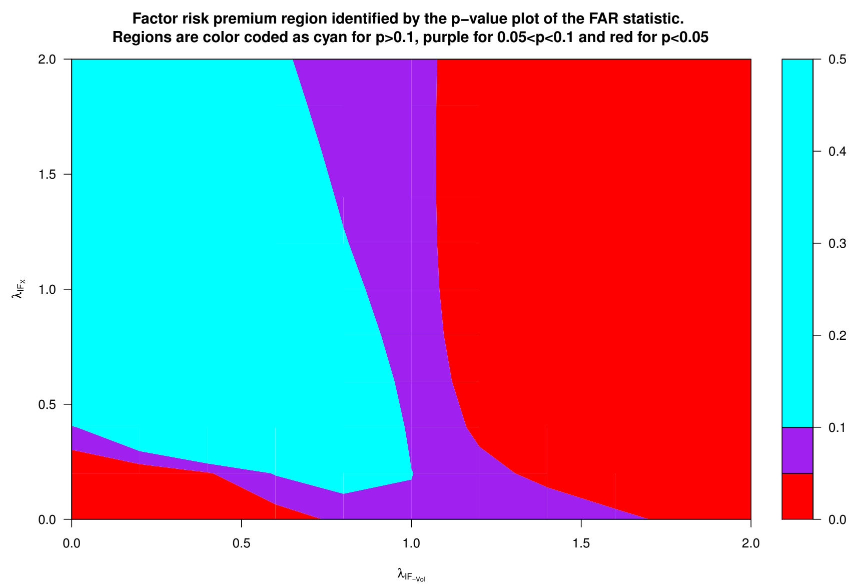

Figure 11

p-value plot of the test of the joint hypothesis of factor pricing together with $(\lambda_{IF_{-Vol}}, \lambda_{IF_X}) = (\hat{\lambda}_{IF_{-Vol}}, \hat{\lambda}_{IF_X})$ using the FAR statistic proposed by Kleibergen (2009). $\lambda_{IF_{-Vol}}$ and λ_{IF_X} are respectively the factor risk premia for the negative volatility and consumption/dividend growth factors.

Since the excess returns of the 25 Fama-French portfolios formed on the basis of size and book to market ratio have a strong factor structure, it is important to use robust test statistics to eliminate the problem of useless factors being identified as useful (a problem forcefully brought out by Kleibergen (2009) and Kleibergen (2010)). Hence, we use the robust test statistics suggested by Kleibergen (2009) to ensure that the factors here are not useless.

We find that these robust test statistics reject the joint hypothesis that $\lambda_{IF_{-Vol}} = \lambda_{IF_X} = 0$ (non-rejection of the hypothesis would indicate that the pricing factors are useless) and do not reject either the hypothesis of factor pricing or that of $\lambda_{IF_{-Vol}} = \hat{\lambda}_{IF_{-Vol}}, \lambda_{IF_X} = \hat{\lambda}_{IF_X}$ for many values of $(\hat{\lambda}_{IF_{-Vol}}, \hat{\lambda}_{IF_X})$ including those estimated using the cross sectional regressions (rejection of this would indicate that the model is rejected by the data). Figure 11 contains the plot of the p-values of the FAR test statistic for many different values of $(\hat{\lambda}_{IF_{-Vol}}, \hat{\lambda}_{IF_X})$. This statistic tests the joint hypothesis of factor pricing and of $\lambda_{IF_{-Vol}} = \hat{\lambda}_{IF_{-Vol}}, \lambda_{F_X} = \hat{\lambda}_{IF_X}$.

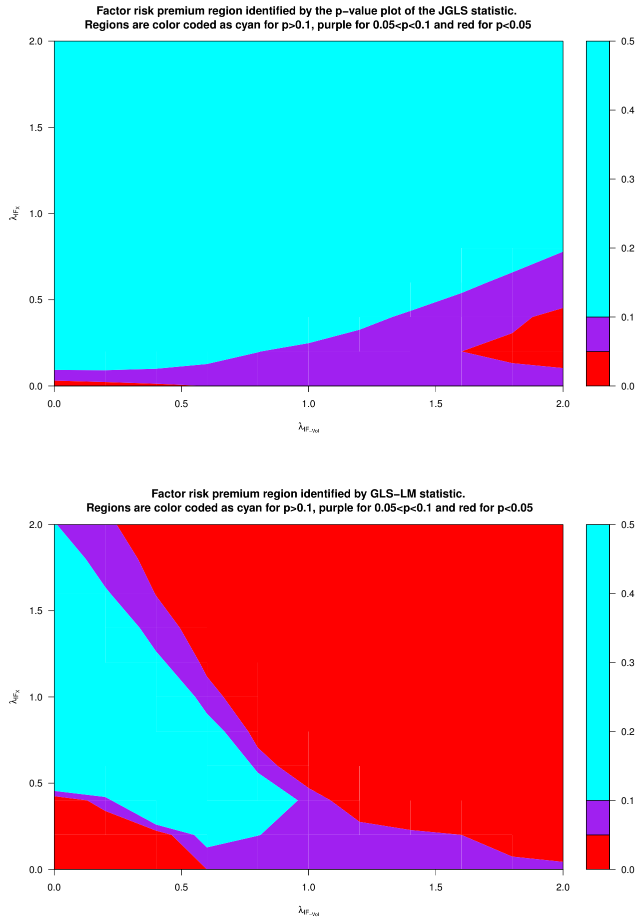

Figure 12

p-value plot of the test of the hypothesis of factor pricing given $(\lambda_{IF_{-Vol}}, \lambda_{IF_X}) = (\hat{\lambda}_{IF_{-Vol}}, \hat{\lambda}_{IF_X})$ using the JGLS and GLS-LM statistics proposed by Kleibergen (2009). $\lambda_{IF_{-Vol}}$ and λ_{IF_X} are respectively the factor risk premia for the negative volatility and consumption/dividend growth factors.

It shows that the joint hypothesis is rejected at $\hat{\lambda}_{IF_{-Vol}} = \hat{\lambda}_{IF_X} = 0$ and also that it is not rejected for many other values of $\hat{\lambda}_{IF_{-Vol}}$ and $\hat{\lambda}_{IF_X}$ including those in table VIII. Further, the region identified by $p > 0.1$ excludes $\lambda_{IF_X} = 0$ but not $\lambda_{IF_{-Vol}} = 0$. This is consistent with the findings using GMM which are analyzed in the next subsection.

The JGLS statistic which tests the hypothesis of factor pricing for a given value of $\lambda_{IF_{-Vol}}$ and λ_{IF_X} is plotted in figure 12. Since it tests a weaker hypothesis, it is not surprising that it rejects fewer values of $\lambda_{IF_{-Vol}}$ and λ_{IF_X} than the FAR statistic. When combined with the GLS-LM statistic, also plotted in figure 12, which tests the hypothesis that $\lambda_{IF_{-Vol}} = \hat{\lambda}_{IF_{-Vol}}, \lambda_{F_X} = \hat{\lambda}_{IF_X}$ given that factor pricing is correct, it gives very similar results to those given by the FAR statistic.

Hence, we can conclude that the robust test statistics show that (21) cannot be rejected. However, they, together with the findings made using GMM, do cast some doubt on the significance of $\lambda_{IF_{-Vol}}$.