Long run consumption risks : Are they there?

Srikant Marakani*

First draft : April 30, 2009
Current version : November 29, 2009

Abstract

Bansal and Yaron (2004) (BY) introduce the concept of "Long Run Consumption Risks" (LRCR). They show that stock and bond returns in an economy exposed to LRCR and populated by agents with Epstein-Zin (1989) preferences will exhibit patterns similar those found in U.S. financial markets. Several follow up studies have examined the LRCR model and find that the data are consistent with it's predictions. Hansen, Heaton, and Li (2008) (HHL) also find empirical support for the presence of LRCR, but of a form somewhat different from the specification studied in BY. In contrast, I find strong evidence against the BY form of the LRCR model using per capita consumption data for the U.S for the period 1929-2008. While the post-1950 consumption data alone cannot reject the LRCR specification in BY, the predictability of per capita consumption growth by the risk free rate and price dividend ratio implied by the model can be rejected for this period. I show that Bansal, Yaron, and Kiku (2007b) reach a different conclusion only because they do not adequately address the bias introduced by time aggregation.

However, I do find that the services component of consumption has highly significant long run autocorrelations which support the fundamental idea of long run risks. Agents may thus believe that overall consumption also has long run risks and this belief could cause the economy to behave as if LRCR existed as pointed out in Hansen and Sargent (2007). Nevertheless, such an approach leads us outside the standard modeling framework.

1 Introduction and Outline

1.1 Introduction

Bansal and Yaron (2004) (BY) and Bansal, Yaron, and Kiku (2007b) show that it is possible to explain apparently anomalous financial market facts within

*Marakani is from the Kellogg School of Management at Northwestern University. I thank Ravi Jagannathan and Ernst Schaumburg for their encouragement, valuable discussions and detailed suggestions and comments. This document is preliminary and all comments are welcome. Do not circulate, cite or quote without the author's permission.

the frictionless efficient markets representative agent framework by introducing the concept of long run consumption risks (LRCR). The BY paper has received wide attention in the literature (see, for example Hansen, Heaton, and Li (2008) Bansal, Dittmar, and Lundblad (2005) Bansal, Dittmar, and Kiku (2009) Hansen and Sargent (2007) Kiku (2006) Dreschler and Yaron (2008) Colacito and Croce (2005) Croce, Lettau, and Ludvigson (2007)). Bansal, Yaron, and Kiku (2007b) find significant measurable predictable LRCR of a form consistent with Bansal and Yaron (2004) in the data and show that these risks are capable of explaining the low risk-free rate and high equity and value premia without requiring a very high relative risk aversion. Hansen, Heaton, and Li (2008) examine several alternative LRCR specifications and find some support, which they admit is fragile, for one that in addition assumes that consumption and corporate earnings are cointegrated.

I closely re-examine the evidence for such measurable predictable LRCR in Bansal, Yaron, and Kiku (2007b) and find that it is quite weak once time aggregation bias (Working (1960), Sims (1971), Geweke (1978), Breeden, Gibbons, and Litzenberger (1989) Christiano (1984), Christiano and Marshall (1987), Christiano, Eichenbaum, and Marshall (1990), Heaton (1993) and others) is taken into account. This result, together with the findings that the variance ratios of consumption growth are inconsistent with the LRCR models in Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007b), and that features of the services component of consumption show clear evidence for the presence of LRCR are the main contributions of this note. The fragility of the evidence for LRCR documented in this note is consistent with the analysis in Hansen, Heaton, and Li (2008), Hansen and Sargent (2007) and Beeler and Campbell (2009). This note goes beyond Beeler and Campbell (2009) in showing that the predictability issues extend to the use of two log price dividend ratios or the market log price dividend ratio together with the real risk free rate as predictors. The use of two predictors is necessary as the BY-LRCR model contains two factors.

Time aggregation is crucial in the Bansal and Yaron LRCR (BY-LRCR) model since it assumes that the agent's decision interval is shorter than the consumption data sampling interval. The importance of this assumption has been emphasized in Bansal and Yaron (2004), Bansal (2007) and Bansal, Yaron, and Kiku (2007b) and the central point of latter, whose analysis I closely examine in this paper, is that the evidence for LRCR is strengthened after accounting for the bias induced by time aggregation. A version of the BY-LRCR model which does not incorporate time aggregation has been analyzed and found to be rejected by the data in Constantinides and Ghosh (2008).

1.2 Outline

I present a short outline of the structure of this note. Section 2, I recall the BY-LRCR model and establish the notation for this paper. In section 3, I analyze the method used to extract the long run consumption risk process in Bansal, Yaron, and Kiku (2007b) and show that the correct handling of time aggregation effects is vital for it's consistency. In section 4, I present a short overview of the data used in this paper. In section 5, I document that the observed variance ratios of annual consumption growth from 1930-2008[1] are inconsistent with the BY-LRCR model. In section 6, I show that the extraction of the long run risk process and, hence, estimation, of the BY-LRCR model in Bansal, Yaron, and Kiku (2007b) is inconsistent due to time aggregation bias and is largely driven by four outliers (1930-1933) in the beginning of the data which closely correspond to the large drop in consumption during the Great Depression.[2] I also show that that a consistent extraction regression which removes the time aggregation bias shows very little predictability and that this is in conflict with the BY-LRCR model for the parameter values estimated in Bansal, Yaron, and Kiku (2007b).[3]

In section 7, I present evidence that there are significant structural breaks in the predictive regressions used in Bansal, Yaron, and Kiku (2007b). This evidence casts significant doubt that the BY-LRCR model can be made consistent with the consumption and dividend data over the period 1930-2008 unless regime shifts are introduced.

I analyze post-1950 data in section 8 and find that the variance ratios alone cannot reject the BY-LRCR model. I show that the anomalously low predictability of the extraction regression continues to hold in this period and that choosing parameter values (such as those in Hansen and Sargent (2007)) consistent with this low predictability leads to counter-factual implications for asset returns.

In section 9, I show that the relative risk aversion value required by the model incorporating cointegration in Hansen, Heaton, and Li (2008) is even higher than the reported value of 20 if per capita rather than total consumption is used, as is the custom in the literature. This result emphasizes the fragility of the evidence for LRCR found in Hansen, Heaton, and Li (2008).

In section 10 I document that the quarterly services consumption data from 1947-2008 unequivocally shows the existence of LRCR within this component. Specifically, services consumption growth exhibits highly significant positive autocorrelations over long lags of up to four years. This is in contrast to consumption growth of nondurables together with services which exhibits little evidence of long run dependence in the data. Hence, one promising avenue for future LRCR research is the pursuit of a more complete model which distinguishes between the different components of consumption. I conclude in section 11. In the appendices, I present detailed discussions of the BY-LRCR model and of the impact of time aggregation on the estimation of time series regressions, particularly those of the type used in the extraction of the LRCR process in Bansal, Yaron, and Kiku (2007b).

[1] the data series favored by Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007b) due to the issues with higher frequency consumption data pointed out in Wilcox (1992)

[2] In this regard, the explanation provided in Bansal, Yaron, and Kiku (2007b) could be an artifact of the disasters model of Barro (2006) and Rietz (1988)

[3] Removal of the outliers or performing a robust estimation to account for them further reduces this predictability.

2 The Bansal-Yaron LRCR model

I provide a brief description of the Bansal-Yaron LRCR model in this section of the paper. The presentation below necessarily closely follows that in Bansal and Yaron (2004).

In this model, the log per capita consumption and per share dividend growth rates g and g_d and their persistent component x are assumed to follow the processes (Bansal and Yaron (2004))

$$\begin{aligned} x_{t+1} &= \rho x_t + \varphi_x \sigma_t e_{t+1} && (1) \\ g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} && (2) \\ g_{d,t+1} &= \mu_d + \phi x_t + \pi_d \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1} && (3) \\ \sigma^2_{t+1} &= \sigma^2 + \nu(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} && (4) \end{aligned}$$

where the shocks e_{t+1}, η_{t+1}, u_{t+1} and w_{t+1} are taken to be independent standard normals[4] for parsimony.

Consumer preferences are assumed to be of the recursive Epstein-Zin (Epstein and Zin (1989)) form. With these preferences, asset returns satisfy

$$E_t[\delta^\theta G_{t+1}^{-\theta/\psi} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \qquad (5)$$

where G is the growth rate of per capita consumption, R_a is the gross return on an asset that provides a dividend of per capita consumption, R_i is the asset return, $0 < \delta < 1$ is the time discount factor, γ is the relative risk aversion, ψ is the inter-temporal elasticity of substitution (IES) and θ is defined to be

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}} \qquad (6)$$

Note that R_a is unobservable but the asset return R_i is. To use this form of the consumer preference relation, it is therefore necessary to relate R_a to observable quantities. This is done by Bansal and Yaron using the standard approximation of Campbell and Shiller (Campbell and Shiller (1988))

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \qquad (7)$$

where lowercase letters refer to the logarithm of the variables represented by the corresponding uppercase letters (i.e. $g_t = \log G_t$, $r_{a,t} = \log R_{a,t}$ etc.), $z = \log(\frac{P}{C})$ is the logarithm of the price to consumption ratio and κ_0 and κ_1 are constants that depend only upon the average value of z which is denoted by $\bar{z}$ (explicitly, $\kappa_1 = \frac{\exp \bar{z}}{1+\exp \bar{z}}$ (Campbell and Shiller (1988))). The analogous approximation for the logarithm of the market return r_m is

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \qquad (8)$$

[4] In Bansal and Yaron (2004), $\pi_d = 0$. Here, I follow the straightforward extension in Kiku (2006).

where z_m is the logarithm of the price to dividend ratio. Since the consumption and dividend growth processes are given by (2) and (3), the dynamics of r_a and r_m can be computed once the dynamics of z and z_m are determined. This is done for Epstein-Zin preferences in Bansal and Yaron (2004) where the solution is found to be

$$\begin{aligned} z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2 && (9) \\ z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 && (10) \end{aligned}$$

The expressions for the A_i and $A_{i,m}$ in terms of the underlying parameters are given in the appendix.

Using the above expressions, Bansal and Yaron (2004) show that the log SDF is given by

$$m_{t+1} = \Gamma_0 + \Gamma_1 x_t + \Gamma_2 \sigma_t^2 - \lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \sigma_t e_{t+1} - \lambda_w \sigma_w w_{t+1} \quad (11)$$

where the expressions for Γ_i are given in the appendix. The market prices of risk corresponding to the different shocks are found to be

$$\begin{aligned} \lambda_\eta &= \gamma && (12) \\ \lambda_e &= \left(\gamma - \frac{1}{\psi}\right) \frac{\kappa_1 \varphi_x}{1 - \kappa_1 \rho} && (13) \\ \lambda_w &= (1-\theta) A_2 \kappa_1 && (14) \end{aligned}$$

The return premium for an asset whose dividend growth process is (3) is then given by

$$\begin{aligned} E_t[r_{m,t+1} - r_{f,t+1}] = &\beta_{m,\eta} \lambda_\eta \sigma_t^2 + \beta_{m,e} \lambda_e \sigma_t^2 + \\ &\beta_{m,w} \lambda_w \sigma_w^2 - \frac{1}{2} \text{var}_t(r_{m,t+1}) \end{aligned} \quad (15)$$

where $r_{f,t+1}$ is the logarithm of the risk-free rate from t to $t+1$ and the factor loadings β_m for the asset are given by

$$\begin{aligned} \beta_{m,\eta} &= \pi_{m,d} && (16) \\ \beta_{m,e} &= \kappa_{1,m} A_{1,m} \varphi_x = \frac{\kappa_{1,m} \varphi_x (\phi - 1/\psi)}{1 - \kappa_{1,m} \rho} && (17) \\ \beta_{m,w} &= \kappa_{1,m} A_{2,m} && (18) \end{aligned}$$

The logarithm of the risk-free rate in the Bansal-Yaron LRCR model is readily derived from the SDF (11) and is given by

$$r_{f,t+1} = A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2 \quad (19)$$

The expressions for $A_{i,f}$ in terms of the underlying parameters are given in the appendix.

The necessity of Epstein-Zin preferences is seen from the factors of $\gamma - 1/\psi$ in (13) and $1 - \theta = \frac{1/\psi - \gamma}{1 - 1/\psi}$ in (14) which are zero for expected utility preferences as they satisfy the relation $1/\psi = \gamma$. Value and small cap stocks within this model are characterized by a high value of ϕ. The return premium commanded by the asset exposure to long run risks is given by $\beta_e \lambda_e \sigma_t^2$ which includes the factor $\frac{(\phi-1/\psi)(\gamma-1/\psi)}{(1-\kappa_1\rho)(1-\kappa_{1,m}\rho)}$. Since $\kappa_{1,m} \sim 0.997$ for reasonable preference parameters and from data (on a monthly time scale), this premium is large when $\rho \sim 1$.

[Table 1 about here.]

I recall the estimates of the time series parameters in Bansal, Yaron, and Kiku (2007b) in table 1 as I use them for quick illustrations of the general implications of the BY-LRCR model.[5]

3 Extraction of the long run risk process

Since the Bansal-Yaron process is specified on a monthly time scale while most of the consumption data that I deal with are on an annual time scale (I follow Bansal and Yaron (2004) in doing this due to the many issues raised regarding the construction of higher frequency data in Wilcox (1992)), I set the time unit to be one year in the following exposition for convenience. The discussion of time aggregation in the appendix follows the same convention.

With this time unit, (19) becomes

$$r_{f,t+\frac{1}{12}} = A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2 \tag{20}$$

where $r_{f,t+\frac{1}{12}} \in \mathcal{F}_t$ is the risk-free rate for an investment from time t to $t + \frac{1}{12}$.

It follows that (20) and (10) can be inverted as pointed out in Bansal and Yaron (2004), Bansal, Yaron, and Kiku (2007b) and Constantinides and Ghosh (2008) to recover the state variables x_t and σ_t^2

$$x_t = \alpha_0 + \alpha_1 r_{f,t+\frac{1}{12}} + \alpha_2 z_{m,t} \tag{21}$$

$$\sigma_t^2 = \alpha_0 + \beta_1 r_{f,t+\frac{1}{12}} + \beta_2 z_{m,t} \tag{22}$$

Note that the same result follows if a price-dividend ratio of another portfolio is used instead of the real risk-free rate. This is generally more convenient since there is no well established way to convert nominal risk-free rates to real ones in the absence of direct data for real risk-free rates.

In Bansal, Yaron, and Kiku (2007b), it is shown that[6]

$$\Delta c_{t+1}^a = \mu + \rho \phi_c^a x_{t-1} + \eta_{t+1}^a \tag{23}$$

$$\Delta c_{t+1}^a = \mu + \rho \phi_c^a (\alpha_0 + \alpha_1 r_{f,t-\frac{11}{12}} + \alpha_2 z_{m,t}) + \eta_{t+1}^a \tag{24}$$

$$x_{t+1} = \rho^{12} x_t + \varphi_x e_{t+1}^a \tag{25}$$

[5] In general, I consider a large set of possible parameter values to check that such implications are not restricted to a point or small region of the parameter space but find that this set provides a good way to get quick estimates of implications of the model.

[6] the equation below is very slightly different in that it uses x_{t-1} rather than $x_{t-\frac{11}{12}}$ but is otherwise essentially the same

where Δc^a is the measured aggregated annual consumption growth. The expressions for ϕ_c^a, η_{t+1}^a and e_{t+1}^a are respectively given in (122), (123) and (124) in the appendix which also contains their derivation.

The above equation is easily generalized to longer term consumption growth

$$c_{t+n}^a - c_t^a = \mu + \rho\phi_c^n x_{t-1} + \eta_{t+n}^n \tag{26}$$

$$c_{t+n}^a - c_t^a = \mu + \rho\phi_c^n(\alpha_0 + \alpha_1 r_{f,t-\frac{11}{12}} + \alpha_2 z_{m,t}) + \eta_{t+n}^n \tag{27}$$

$$x_{t+n} = \rho^{12n} x_t + \varphi_x e_{t+1}^n \tag{28}$$

The expressions for ϕ_c^n, η_{t+n}^n and e_{t+n}^n are respectively given in (137), (138) and (139) in the appendix.

The process $\rho\phi_c^a x$[7] is therefore consistently extracted by estimating the regression

$$\Delta c_{t+1}^a = \beta X_{t-1} + \eta_{t+1}^a \tag{29}$$

where $X_{t-1} = (1, r_{f,t-1+\frac{1}{12}}, z_{m,t-1})$, $E_{t-1}[\eta_{t+1}^a] = 0$ and $\text{Cov}(\eta_t^a, \eta_{t+k}^a) = 0$ for $|k| > 1$. β is estimated consistently using simple OLS since $E[\eta_{t+1}^a | X_{t-1}] = 0$. The process $\rho\phi_c^a x_{t-1}$ is then estimated as $\beta X_{t-1} - \mu$.

Examination of the extent to which consumption growth is predicted by the real risk-free rate and log price-dividend ratio can be done using the above regression.

As noted in Bansal, Yaron, and Kiku (2007b), (23) can also be expressed as

$$\Delta c_{t+1}^a = \mu + \rho^{-11}\phi_c^a x_t^a + \tilde{\eta}_{t+1}^a \tag{30}$$

$$x_{t+1}^a = \rho^{12} x_t^a + \varphi_x e_{t+1}^a \tag{31}$$

where $E_{t-1}[\tilde{\eta}_{t+1}^a] = 0$. Bansal, Yaron, and Kiku (2007b) use this equation together with (21) to estimate x by performing the regression

$$\Delta c_{t+1}^a = \beta X_t + \omega\eta_t^a + \eta_{t+1}^a \tag{32}$$

However, as shown in the appendix, this extraction procedure is inconsistent since, under the null, ω is time varying and $E_t[\eta_{t+1}^a | X_t] \neq 0$.

4 Data

In this section, I describe the data used in this paper. Consumption data are obtained from the National Income and Product Accounts (NIPA) tables available at the BEA website. Real annual per capita consumption is defined to be the sum of the aggregate annual nominal consumptions of nondurables and services divided by the NIPA estimate of the mid-year population and deflated by the overall personal consumption deflator. The first difference of the logarithm of this series is defined to be the annual consumption growth. The implicit price deflator (the weighted average of the nondurables and services price deflators) is

[7] Clearly, $\rho\phi_c^n x$ can also be extracted in a similar manner

also commonly used to convert nominal consumption to real and I have checked that my results are robust to it's use. In the analysis of the Hansen, Heaton and Li model (Hansen, Heaton, and Li (2008)) and the autocorrelation structure of consumption growth, quarterly consumption data from NIPA are used with the consumption and its growth being defined in exactly the same manner as for the annual data.

The proxy used for the nominal risk-free rate is the Fama 3 month T-bill rate taken from CRSP. As described in the analysis below, this is converted to a real rate using several measures such as the realized, past and expected inflation over different horizons as measured by CPI. The realized and expected CPI data are taken from CRSP and the Livingston survey results available at the website of the Federal Reserve Bank of Philadelphia. The expected inflation is defined to be twice (for annualization) the difference between the logarithms of the median expected CPI six months into the future and the current CPI.[8]

The market proxy used is the CRSP value-weighted index of all stocks on the NYSE, AMEX and NASDAQ. The construction of the size and book-to-market portfolios is as in Fama and French (1993). The growth and value portfolios denote the bottom and top BE/ME deciles respectively. The data for these portfolios are taken from Ken French's website. Monthly dividends are calculated using the difference between the returns of the dividend reinvested and non-reinvested portfolios. The price-dividend ratio is obtained by dividing the real price of the non-reinvested portfolio by the sum of the lagged twelve real monthly dividends to account for the pronounced seasonality of the dividend series. For this purpose, the prices and dividends are deflated by the CPI.

When analyzing the Hansen, Heaton and Li model (Hansen, Heaton, and Li (2008)), the programs and data (with the exception of consumption) from the data appendix on Nan Li's website are used. It should be noted that since the consumption series in the appendix is not per capita, it is replaced with per capita consumption.[9]

5 Variance ratios and the BY-LRCR model

I define the variance ratio statistics (Campbell, Lo, and MacKinlay (1997)) for a time series as

$$\bar{\mathrm{VR}}(q) = \frac{\bar{\sigma}_c^2(q)}{q\bar{\sigma}_a^2} \tag{33}$$

[8] Since shorter term expected inflation data only exists for the last few years, the shortest available term for the overall series (for which data is available from 1946) is used.

[9] The earnings to consumption ratio is not changed since both variables are now per capita.

where

$$\bar{\sigma}_c^2(q) = \frac{(N-1)\sum_{k=q+1}^{N}(c_k - c_{k-q} - q\hat{\mu})^2}{(N-q)(N-q-1)} \tag{34}$$

$$\bar{\sigma}_a^2 = \frac{1}{N-2}\sum_{k=2}^{N}(c_k - c_{k-1} - \hat{\mu})^2 \tag{35}$$

$$\hat{\mu} = \frac{c_N - c_1}{N-1} \tag{36}$$

This definition is somewhat different from the usual one where the data is truncated to a multiple of q when calculating the qth variance ratio. I feel that this is better as it makes more use of the data. I have verified that the following results are not materially affected by this choice and that using the more usual definition leads to the same conclusions.

The real per capita consumption and its logarithm are plotted in figure 1. The variance ratio statistics $\bar{\text{VR}}(i)$ for the latter series are plotted in figure 2. This set of observed variance ratio statistics is very unusual. While the strong peak observed at short lags is consistent with the short term positive autocorrelation introduced by time aggregation (Working (1960)) and the LRCR hypothesis, the quick decrease as the lag increases from about 3 to 7 or 8 is very remarkable. This indicates that in the longer term, the effect of long run risks is reversed to some extent and it is possible that the data can be described as having long run risks but also being trend-stationary. I have thus investigated a trend-stationary version of the LRCR model in Marakani (2009) and find mixed support for it.

[Figure 1 about here.]

I now investigate the possibility that this striking set of variance ratio statistics has been produced by the BY-LRCR consumption process (2). I define the test statistic

$$t^2 = (\text{VR} - \mu)^T \Sigma^{-1} (\text{VR} - \mu) \tag{37}$$

where VR is the column vector of variance ratios from lags 2 to 10 and μ and Σ are the mean and covariance of VR under the null. Asymptotically, variance ratios are normally distributed (Campbell, Lo, and MacKinlay (1997)) and therefore $t^2 \sim \chi^2(9)$.

Due to the short length of the available consumption series, I use Monte Carlo methods to estimate μ, Σ and the distribution of t^2. I create simulated consumption paths using the null of the BY-LRCR consumption process specified in (1), (2) and (4) with the aggregate annual consumption being calculated as the sum of the simulated monthly consumptions over the year. I set the initial values of Δc, x and σ in the simulated paths to their mean values but discard the first 500 simulated time steps so that the effect of these initial values on the results is minimized. The next 960 time steps (80 years) of each simulated path (the size of the observed data sample) is used to calculate the simulated test statistic for that path. The implied p-value of the test statistic is then computed using the number of simulated test statistics below the observed value in the usual manner.

I first present the results of this procedure for the parameter estimates in Bansal, Yaron, and Kiku (2007b). The estimates of the mean and covariance of VR generated from 100,000 sample paths (or 9.6×10^7 months) imply a value of 76.66 for the test statistic t^2. The asymptotic p-value is a negligible 7.4×10^{-13}.[10] Of the 100,000 simulated test statistic values , 7 exceed the observed value indicating an actual p-value of about 7×10^{-5}.[11] The difference between the two p-values is a result of the short length of the consumption time series. The effect of this is seen in more detail in the plot of the test statistic's asymptotic and Monte Carlo densities in figure 4. The Monte Carlo density is more positively skewed and has a fatter tail than the asymptotic density.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

Keeping in mind that the parameters in Bansal, Yaron, and Kiku (2007b) were estimated with significant error, the above test was repeated for a large grid of points in the parameter space. Since the values of μ and σ do not affect the variance ratios (provided σ_w is scaled appropriately), the following four dimensional hyper-grid was chosen for this purpose

$$(\varphi_x, \rho, \sigma_w, \nu) = \left(0.015i, 1 - \frac{1}{20j}, 10^{-6}k, 1 - \frac{1}{20l}\right), 1 \leq i, j, k, l \leq 5 \qquad (38)$$

At each point in this hyper-grid, 10,000 simulated BY-LRCR paths were used to estimate the mean and covariance of VR and calculate one thousand simulated values of t^2. These estimates were then used together with the observed variance ratios to calculate the observed value of t^2 for each point. The implied p-value for each point in the hyper-grid was then calculated in the usual manner.[12] The maximum implied p-value thus obtained over the entire hyper-grid was 0 (since only 1000 simulated values were used, this only means that $p < 0.001$ at each point in the grid).[13] In other words, the BY-LRCR consumption process is unable to explain the observed variance ratios of the consumption data for a large range of reasonable parameter values.

The variance ratios observed in the data are more consistent with there being no long run risks (in other words, with $\varphi_x = 0$) as can be seen from figure 3.[14]. However, the observed value of t^2 under the null of $\varphi_x = 0$ (47.1) is still

[10] The corresponding values when the implicit deflator is used are 57.95 and 3.3×10^{-9}.
[11] The corresponding value when the implicit deflator is used is 2.4×10^{-4}
[12] Only 1000 simulated values of t^2 were used for this purpose to save computer time.
[13] The result is unchanged when the implicit deflator is used.
[14] The mean variance ratio for non-zero lags is still greater than one due to the MA(1) type autocorrelation structure introduced by time aggregation Working (1960)

very high and the p-value implied by Monte Carlo is still very low (0.0003). Hence, the other assumptions made in the BY-LRCR model (such as a constant μ and standard normal innovations) need to be investigated. I am currently performing such an investigation.

[Figure 5 about here.]

The relation

$$\bar{\mathrm{VR}}(q) = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\hat{\rho}_k \tag{39}$$

where $\hat{\rho}_k$ is the sample autocorrelation with lag k, holds asymptotically under fairly mild assumptions (Campbell, Lo, and MacKinlay (1997)). However, I find that for annual consumption growth, the two expressions give rather different results. This result is very clearly seen in figure 5 and indicates the existence of long term deviations from stationarity in this series. These deviations can be in the form of large jumps as in Rietz (1988) and Barro (2006) or structural breaks in the mean and volatility of consumption growth as in Bai (1997) and Bai and Perron (1998). Since my preliminary analysis shows evidence for structural breaks in the mean and volatility of consumption growth in the 1930s and 1940s, I analyze post-1950 data later in the paper and find that the relation (39) holds fairly well for this period (see figure 6).

[Figure 6 about here.]

6 Time aggregation and the extraction of x

The BY-LRCR model implies that the regression used to extract the persistent predictable component of consumption growth x should have a high r^2. This can be seen analytically using (24) which gives

$$E_t[\Delta c^a_{t+1}] = \mu + \rho\phi^a_c x_{t-1} = \mu + \alpha_0 + \alpha_1 r_{f,t-\frac{11}{12}} + \alpha_2 z_{m,t-1} \tag{40}$$

and (in the absence of heteroskedasticity)

$$\mathrm{Var}(E_t[\Delta c^a_{t+1}]) = (\rho\phi^a_c)^2\mathrm{Var}(x) = \frac{(\rho\phi^a_c\varphi_x\sigma)^2}{1-\rho^2} \tag{41}$$

so that the expected r^2 is

$$r^2 = \frac{\mathrm{Var}(E_t[\Delta c^a_{t+1}])}{\mathrm{Var}(\Delta c^a_{t+1})} = \frac{\frac{(\rho\phi^a_c\varphi_x)^2}{1-\rho^2}}{\sum_{j=0}^{22}\tau_j^2\left(1+\varphi_x^2\left(\sum_{k=1}^{j}\rho^{k-1}\right)^2\right) + \frac{(\rho\phi^a_c\varphi_x)^2}{1-\rho^2}} \tag{42}$$

For $\varphi_x = 0.042$ and $\rho = 0.982$, the calibration in Bansal, Yaron, and Kiku (2007b), this gives an expected r^2 of about 32.8%. While this result is not directly applicable since heteroskedasticity is present in the the BY-LRCR model, Monte Carlo simulations indicate similar levels of predictability even when heteroskedasticity is taken into account. Monte Carlo simulations using the parameter values in Bansal, Yaron, and Kiku (2007b) give a median adjusted r^2 of 27.7% with a 5th percentile of 6.4% and a 95th percentile of 50.1%. Bansal, Yaron, and Kiku (2007b) report that their Monte Carlo simulations give a much higher expected r^2 between 25% (5th percentile) and 65% (95th percentile) with a median value of 43%.[15]

6.1 Extracting x using the real risk-free rate and the log price-dividend ratio

Using (29), I attempt to extract x as in Bansal, Yaron, and Kiku (2007b). I first do so using the real risk-free rate and the log market price-dividend ratio. The real risk-free rate is obtained by converting the nominal risk-free rate to a real one using various procedures. The best way to do this would be to use the representative consumer's expected inflation. However, since expected inflation data is only available from the mid 1940s[16], this is not feasible. I therefore carry out the extraction procedure using two different methods of calculating the real risk-free rate and then compare the resulting findings. The first method assumes that inflation is perfectly predictable so that the nominal risk-free rate is converted to a real rate by using realized future inflation. Inflation on an annual and quarterly rather than monthly basis is used for this purpose since monthly inflation is seriously affected by seasonality and the rounding of the CPI series, particularly early in the data sample. The second method assumes that the agent's expected inflation is the realized past inflation following Gultekin (1983) and the nominal risk-free rate is converted to a real rate using lagged quarterly and annual inflation. The second method is not unreasonable since inflation is persistent but ignores other information that the agents might possess while the first method incorporates a significant look-ahead bias. It is likely that the representative consumer's expected inflation is somewhere between the perfect foresight and pure hindsight assumptions that are used here.

Table 2 summarizes the results of the extraction regressions which do not assume any particular form for the residual autocorrelation (i.e. simple OLS). Table 3 summarizes the results of the extraction regressions which assume a MA(1) form for the residual autocorrelation (the expected structure due to time aggregation). Table 4 summarizes the results of the inconsistent regression (32) so that they can be compared with the results of the consistent regressions. The adjusted r^2 values obtained for the consistent regressions are *much* lesser than the 35% reported in Bansal, Yaron, and Kiku (2007b) and are also smaller than the 5th percentile values obtained from our Monte Carlo estimates.[17] Only the

[15] It is possible that the value reported in Bansal, Yaron, and Kiku (2007b) is for the inconsistent regression using singly lagged predictors since the expected r^2 obtained from our Monte Carlo simulations are consistent with these reported values.

[16] The Livingston survey started in 1946.

[17] I did not receive replies to our requests to Bansal, Yaron and Kiku for their data and I could not therefore check the source of this large discrepancy.

adjusted r^2 value for the inconsistent regression where realized annual inflation is used to convert the nominal risk-free rate into a real one comes close it. This regression, is, however, affected by time aggregation as well as look-ahead bias.

Similar points regarding the predictability of consumption growth using a *single* price-dividend ratio have been made in Bui (2007) and in Beeler and Campbell (2009). I am, however, not aware of any study which has pointed out that the extraction regression in Bansal, Yaron, and Kiku (2007b) shows much less predictability than reported once time aggregation is taken into account.

I also find that the above regressions are susceptible to being largely, if not entirely, driven by four outliers (1930-1933) corresponding to a possible break in the mean of the consumption growth (or a jump as interpreted by Rietz (1988) and Barro (2006)). The effect of these outliers on the estimate of the coefficient of the real risk-free rate in the extraction regression can be seen very clearly in figures 7 and 8. The coefficients of the regressions are also unstable and the test for structural breaks in Bai and Perron (1998) using the algorithm in Bai and Perron (2003) as implemented in Zeileis, Leisch, Hornik, and Kleiber (2002), Zeileis, Kleiber, Krämer, and Hornik (2003) and Zeileis (2006), shows significant evidence for a change in the value of the coefficients at about 1942.

The effect of the outliers is evident from the significant reduction in the r^2 of the predictability regression in Bansal, Yaron, and Kiku (2007b) when the data from 1930-1933 is dropped (note that this regression generally provides spuriously positive evidence for predictability in the presence of time aggregation). Dropping the real risk free data for these years is also reasonable as deflation constrains its value in a manner not captured by the long run risk model and because there was a lot of uncertainty regarding inflation during this period. The r^2, after dropping these points, is reduced to 9.4% or 7.2% depending on whether realized or lagged inflation is used to convert the risk free rate to a real. These values are well below the 5th percentile of 25% reported for the Monte Carlo simulations in Bansal, Yaron, and Kiku (2007b). This fact, together with the fact that the r^2 is supposed to be constant over time constitutes significant evidence against the long run risk model proposed in Bansal, Yaron, and Kiku (2007b) even if time aggregation is ignored.

I note that the results of these extraction regressions do not significantly depend on the method used to convert the nominal risk-free rate to a real one. In all the cases, there is evidence that contemporaneous correlation between innovations in the regressors and consumption growth gives rise to high levels of spurious predictability in the inconsistent regressions involving singly lagged variables and that a few outliers at the beginning of the consumption data drive most of the results.

With regard to asset pricing implications, I note that performing the extraction of the latent state variable x in a manner consistent with time aggregation results in a process whose variance is too small given the assumed value of φ_x in Bansal, Yaron, and Kiku (2007b). Lowering the value of φ_x to make it consistent with the extracted process x also lowers the implied equity premium, or in other words, increases the relative risk aversion estimate given the return data. Hence, ignoring time aggregation biases the relative risk aversion estimates downwards rather than upwards as claimed in Bansal, Yaron, and Kiku (2007b). The parameters suggested in Bansal, Yaron, and Kiku (2007a) also suffer from this problem, though to a smaller extent, and do not resolve the predictability issue after 1933 when both predictors are used.

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

6.2 Extraction using two (or more) price-dividend ratios

To remove complications regarding the predictability of inflation, the result that the log price-dividend ratio is a linear function of the long run risk and volatility processes (namely x and σ^2) (10) is used for two different portfolios to perform an estimation similar to that in the previous subsection. The market portfolio (defined as the value weighted NYSE/NASDAQ/AMEX index from CRSP) and the value stock portfolio (defined as the portfolio composed of the the top 10% of stocks ranked by their book to market ratio as defined in Fama and French (1992)) are used for this purpose. The results of this estimation are summarized in table 5. The adjusted r^2 is found to be -0.014 indicating no predictability at all.

Similar results are also found when the price-dividend ratios of other portfolios or the first three principal components of the log price-dividend ratios of the BE/ME decile portfolios (which account for over 98.5% of the variance) are used in this estimation. The results are summarized in table 6. Using longer term consumption growth does not help and the r^2 for the corresponding regression using three year annual consumption growth is only 0.014 (the adjusted r^2 being -0.026). This latter result implies that the addition of state variables to the BY-LRCR model will be unable to account for the non-predictability of consumption growth using price-dividend ratios of various portfolios of stocks.

[Table 5 about here.]

[Table 6 about here.]

7 Structural breaks in the above estimations

Given the lack of stationarity in the consumption series indicated by the different results obtained using the standard calculation and (39) for the variance ratios of the consumption data, the test for structural breaks in (Bai and Perron 1998) using the algorithm in (Bai and Perron 2003) as implemented in (Zeileis, Leisch, Hornik, and Kleiber 2002), (Zeileis, Kleiber, Krämer, and Hornik 2003) and (Zeileis 2006) is performed for the extraction regressions. For the extraction regression

$$E_t[\Delta c_{t+1}] = \mu + \alpha_0 + \alpha_1 r_{f,t+1} + \alpha_2 z_{m,t} \tag{43}$$

this gives strong evidence of a break at about 1942 (BIC with break : -406.39, without break : -363.54) with a strong drop in predictive power after the break (most of the high predictive power in the pre-1942 series is attributable to a couple of outliers).

There is also evidence of a structural break at about the same time in the extraction regression where the market and value price-dividend ratios are used. The break is estimated to be at 1941 (BIC with break : -361.11, without break : -361.04) with the small difference in BIC probably due to the low predictability using these price-dividend ratios in both periods.

Similar results also hold when the first two or three principal components of the log price-dividend ratios of the six Fama-French portfolios based on size and BE/ME are used.[18] The breaks for both these regressions occur at 1941 with the BIC values with and without breaks for the two component regression being -377.42 and -364.54 respectively. The corresponding BIC values for the three component regression are -368.79 and -360.19 respectively. I note that the significance of the structural breaks only increases if I perform the test using the regressions with singly lagged variables.

These results provide strong evidence that the parameters of the BY-LRR model were not constant over the entire sample but changed significantly at around 1941. Significant evidence for breaks in the consumption growth volatility, earnings to consumption ratio and the dividend growth of value stocks at the same time is also found in the data using the same method. Hence, a possible fruitful avenue for future research is the exploration of a LRCR model that incorporates infrequent but large changes in the parameters.

8 Post-1950 data and the the BY-LRCR model

The variance ratio statistics for post-1950 consumption growth plotted in figures 9 and 10 do not exhibit the large decrease after 7 to 8 lags found for the entire series (there is a much milder decrease but that is not inconsistent with the the BY-LRCR consumption process) and the BY-LRCR process cannot be rejected using them. When the null is taken to be the BY-LRCR process estimated in Bansal, Yaron, and Kiku (2007b), the value of t^2 is found to be 10.28 and when the null is taken to be the process with no long run risks, it's value is found to be 8.87. Hence, both processes are found to be consistent with the variance ratios in the data. The relation (39) also approximately holds and this is consistent with a lack of significant structural breaks in the post-1950 consumption series. The variance ratio statistics and the corresponding weighted sum of autocorrelations (39) are plotted in figure 6.

[18] We cannot use the BE/ME decile portfolios for this purpose since the price-dividend ratio of B9 is undefined for 1933 since it had no dividends during that year.

The autocorrelations of post-1950 annual consumption growth are plotted in figure 11. As pointed out in Beeler and Campbell (2009), they are negative for all lags from 2 to 10 and indicate a lack of a long term component in consumption growth. However, this is only an indication and these autocorrelation values are found to be not jointly significantly different from zero using a Ljung-Box type test.

[Figure 9 about here.]

[Figure 10 about here.]

[Figure 11 about here.]

The high level of predictability of consumption growth by the real risk-free rate and a price-dividend ratio or two price-dividend ratios that is implied by the BY-LRCR model (for reasonable parameter values) is not present in the post-1950 data as well. The results of the OLS extraction regressions for post-1950 data are summarized in table 7. The corresponding results of the regressions where the expected MA(1) residual autocorrelation structure is taken into account are summarized in table 8. As explained in section 4, the annualized median expected six month inflation from the Livingston survey is used to convert the nominal risk-free rate to a real one for the first estimation[19] and the price-dividend ratios of the value and market portfolios are used for the second estimation (use of the price-dividend ratios of other portfolios gives similar results).

[Table 7 about here.]

[Table 8 about here.]

It is evident from table 7 that the predictability of consumption growth is much smaller than that implied the BY-LRCR model. There is some predictability in the regression involving the two price-dividend ratios after residual correlation is taken into account but even this of a much smaller extent than that implied by the BY-LRCR model, particularly for parameter values close to those estimated in Bansal, Yaron, and Kiku (2007b).

(42) and Monte Carlo simulations indicate that such low adjusted r^2 values can result from the BY-LRCR model only if φ_x is very small (less than 0.01). In that case, the long run risk process must be very persistent (with a half-life of at least ten years for it's shocks) to generate a reasonable equity premium with a relative risk aversion of about 10, the value favored in Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007b).[20] Such a highly persistent process

[19] Shorter term expected inflation would have been better but such data is only available for only a small sub-sample of the post-1950 period.

[20] Another possibility which would also explain the results is that the market dividend growth rate was only mildly driven by x (i.e., ϕ is low) but the model would then be unable to explain the equity premium.

implies higher predictability of longer term consumption growth as can be seen from the equivalent of (42) for (27) or from the general arguments for persistent predictors in Campbell, Lo, and MacKinlay (1997). This higher predictability for longer term consumption growth is indeed found in the data. However, the relation between the x that is thus extracted and the dividend growth of various portfolios is precisely the opposite of what is expected from the model.

When x is estimated using five year consumption growth with the real risk-free rate and logarithm of the market price dividend ratio as predictors, the estimated values of ϕ are -17.27 (Newey-West std. error of 6.48) for the value portfolio and 7.12 (Newey-West std. error of 8.60) for the growth portfolio. When the logarithm of the market and value price-dividend ratios are used as predictors, the estimated values of ϕ for the value and growth portfolios are -28.25 (Newey-West std. error of 10.84) and 10.61 (Newey-West std. error of 14.52) respectively. With these estimates, the model predicts a large *negative* value premium.[21]

9 Other studies of long run risk models

Another significant and influential study of long run risk models is the one carried out in Hansen, Heaton, and Li (2008). In this study, Hansen, Heaton, and Li (2008) use a VAR of the earnings to consumption ratio and consumption growth to estimate the predictable long run component of consumption. They use this estimate together with the dividend dynamics of the market and book to market ratio sorted portfolios to study the possibility of explaining the equity and value premium with it. They find marginal but not strong evidence for the presence of long run risks and conclude that, while the identified long run risk component is capable of explaining the value premium with a risk aversion of about 20, it is incapable of explaining the equity premium with a risk aversion of less than 100. Since their study uses total rather than per capita consumption growth, as is customary in the literature, I repeat it using per capita figures. With this modification, I find that the risk aversions required by the model to explain the value and equity premium are about 30 and over 100 respectively.[22] Further, I find that the causality implied by the results of the VAR in Hansen, Heaton, and Li (2008) is the reverse of that expected from the long run risk model. From the results of the VAR (using total consumption, as in Hansen, Heaton, and Li (2008)), I find that the earnings to consumption ratio (the predictor of consumption growth used in the analysis) does not Granger cause consumption growth and that the p-value of the F-test for this Granger causality is 0.482. In contrast, I find that consumption growth does Granger cause the earnings to consumption ratio and that the p-value of the F-test for this Granger

[21] The extraction regression using long term consumption growth is susceptible to the well known problems that arise when the regressors are persistent, particularly when two price-dividend ratios are used as the predictors.

[22] Hansen, Heaton, and Li (2008) are aware of the issue with regard to per capita data and have noted that, when per capita data is used, the estimated long run component using their approach is reduced by about 20% (private communication).

causality is 2.2×10^{-4}.[23] From these findings, I conclude that the evidence for long run risks documented in Hansen, Heaton, and Li (2008) is very fragile indeed. However, I also note that Hansen, Heaton, and Li (2008) themselves acknowledge that their results do not provide any conclusive evidence for the presence of long run risks.

Another interesting study of the predictability of consumption growth implied by long run risk models is by Bansal, Yaron, and Kiku (2007a). However, this study limits itself to the analysis of the predictability of consumption growth by the log market price dividend ratio alone. Hence, it's results are not directly applicable here and my Monte Carlo results show that the long run risk models are incapable of explaining the low predictability of consumption growth using two log price dividend ratios or the log market price dividend ratio and the real risk free rate even if the parameters are changed to the values suggested in this study.

10 Long run risks in components of consumption

As noted earlier in this paper, annual consumption growth over the period 1950-2008 exhibits many negative sample autocorrelation coefficients (a point particularly noted in Beeler and Campbell (2009)). However, I find that the values of Portmanteau type autocorrelation test statistics (similar to Ljung-Box but calculated without the first autocorrelation coefficient to account for time aggregation) are insignificant for this series. This is probably because annual data only provides a short series due its coarseness. In contrast, I find that short term autocorrelations are significant for the available quarterly consumption data from 1947-2008 (thus confirming the results of Carroll, Slacalek, and Sommer (2008)) and that short as well as long term autocorrelations are very highly significant for the services component of consumption.

[Figure 12 about here.]

[Figure 13 about here.]

Quarterly consumption growth shows significant short term autocorrelations (Carroll, Slacalek, and Sommer (2008)) even after accounting for time aggregation. I plot these autocorrelations in figure 12. They indicate a short half-life of only about six months for the "long term" shocks to consumption growth and thus do not provide positive evidence for LRCR unless the LRCR time scale is changed to a value significantly different from that assumed in the literature.

[23] The Wald test for instantaneous causality between the two shows significant instantaneous causality between the two, but this result is not surprising since consumption and earnings are not perfectly cointegrated.

I examine the significance of these autocorrelations by defining the following Ljung-Box type test statistic for quarterly consumption growth

$$L(m) = \sum_{k=2}^{m} n(n+2)\frac{\hat{\rho}_k^2}{n-k} \tag{44}$$

where n is the number of sample data points and $\hat{\rho}_k$ is the kth sample autocorrelation coefficient. I start the sum from the second autocorrelation coefficient to eliminate time aggregation complications. Under the null of i.i.d consumption growth, $L(m) \sim \chi^2(m-1)$ asymptotically. Using this asymptotic distribution, I find that $L(m)$ is significant at the 1% level for $m \leq 5$. Autocorrelations for lags greater than 5 quarters are not significant and the p-value for $L(m)$ increases as m is increased beyond 5. $L(m) - L(5)$ is not significant even at the 10% level for any value of $6 \leq m \leq 15$.

This picture changes significantly when only the services component of consumption is examined. For this purpose, I define the real per capita services consumption as the nominal seasonally adjusted aggregate services consumption divided by the NIPA estimate of the mid-quarter population and deflated by the services consumption deflator. The relevant data series for the computation of this quantity are obtained from the NIPA tables on the BEA web site. The autocorrelation structure of real per capita services consumption growth is plotted in figure 13. The short term autocorrelations are much less significant for this series with $L(m)$ being significant at the 5% level but insignificant at the 1% level for $m \leq 5$. However, the autocorrelations at lags 6 to 9 are positive, large and highly significant and $L(m)$ is significant even at the 0.1% level for $8 \leq m \leq 20$. This shows that a highly significant long term component exists for quarterly services consumption growth. Further, the asymptotic statistics used here are conservative. This is because, in finite samples and under the null of i.i.d consumption growth, the mean of the autocorrelation is negative (for all lags) and it's variance is lower than the asymptotic value (Kan and Wang (2008)).

Hence, while only a short term component of consumption growth can be positively identified when the total per capita consumption of nondurables and services is used as in Bansal and Yaron (2004) and Bansal, Yaron, and Kiku (2007b), a significantly longer term component can be identified when per capita consumption of services is used instead. I thus believe that it would be fruitful to analyze a version of the LRCR model which distinguishes between the different components of consumption, both in terms of their long term properties and their relation to the agent's preferences. I am currently working on making and testing such a model.[24]

[24] Extending the assumption that the utility is additively separable over services and nondurables on the one hand and durables on the other to services on the one hand and nondurables and durables on the other does not solve the problem. This is because replacing the per capita consumption of nondurables and services with the per capita consumption of services gives estimates of ϕ (for an x extracted from twenty quarter services consumption growth using the market and value price-dividend ratios) which are significantly higher for

11 Conclusion

In this paper, I show that the time series specification for per capita consumption growth in Bansal and Yaron (2004) can be rejected using the available annual data from 1929-2008 I also show that the relatively high predictability of consumption growth by the price-dividend ratio and real risk-free rate implied by the BY-LRCR model is not found in the data for both the 1929-2008 and 1950-2008 periods once time aggregation effects are taken into account (similar issues regarding predictability but only using the price-dividend ratio have been noted earlier in Bui (2007) and in Beeler and Campbell (2009)).

I find that the LRCR model incorporating a cointegration assumption in Hansen, Heaton, and Li (2008) requires even higher risk aversion values than postulated therein when per capita rather than total consumption figures are used, as is customary in the literature. I also argue that changing the parameter values to those suggested in Bansal, Yaron, and Kiku (2007a) does not overcome the predictability issue when two log price dividend ratios or one log price dividend ratio and real risk free rate are used as the predictors. Such a change, however, does overcome the predictability issues raised in Beeler and Campbell (2009) as noted in Bansal, Yaron, and Kiku (2009).

While I find little positive evidence for a long run component in the total per capita consumption of nondurables and services, I find strong evidence for it in the per capita consumption of services. A promising avenue for future research which I am thus exploring is the generalization of the preference relation and consumption process in the LRCR model to account for the variation among these different components.

I also note that even if LRCR do not exist, people might behave as if they do due to uncertainty or ambiguity aversion (as pointed out in Hansen and Sargent (2007)). This is particularly plausible given that agents' beliefs might be affected by the evidence for LRCR found in the services component of consumption and that dividend and earnings growth are strongly affected by consumption shocks over several quarters (Bansal, Dittmar, and Lundblad (2005) Bansal, Dittmar, and Kiku (2009) Hansen, Heaton, and Li (2008)). Further, as shown in Malloy, Moskowitz, and Vissing-Jørgensen (2009), stockholder consumption does have LRCR and that those risks are sufficient to explain the cross sectional returns of many assets.

References

Bai, Jushan, 1997, Estimation of a change point in multiple regression models, *Review of Economics and Statistics* 79, 551–563.

——— , and Pierre Perron, 1998, Estimating and testing linear models with multiple structural changes, *Econometrica* 66, 47–78.

growth stocks (0.45 with a standard error of 1.53) than for value stocks (-4.48 with a standard error of 2.43). Similar results hold for LRCR processes extracted using other price-dividend ratios.

——— , 2003, Computation and analysis of multiple structural change models, *Journal of Applied Econometrics* 18, 1–22.

Bansal, Ravi, 2007, Long run risks and financial markets, *The Review of the St. Louis Federal Reserve Bank 2007* 89, 1–17.

——— , Robert F. Dittmar, and Dana Kiku, 2009, Cointegration and consumption risks in asset returns, *Review of Financial Studies* 22, 1343–1375.

Bansal, Ravi, Robert F. Dittmar, and Christian Lundblad, 2005, Consumption, dividends, and the cross-section of equity returns, *Journal of Finance* 60, 1639–1672.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

——— , and Dana Kiku, 2007a, A note on the economics and statistics of predictability: A long run risks perspective, Working Paper.

——— , 2007b, Risks for the long run : Estimation and inference, Working paper.

——— , 2009, An empirical evaluation of the long-run risks model for asset prices, Working paper.

Barro, Robert, 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Beeler, Jason, and John Y. Campbell, 2009, The long-run risks model and aggregate asset prices : An empirical assessment, Working paper, Harvard University.

Breeden, Douglas, Michael Gibbons, and Robert Litzenberger, 1989, Empirical Test of the Consumption-Oriented CAPM, *Journal of Finance* 44, 231–262.

Bui, Minh P., 2007, Long-run risks and long-run predictability : A comment, Unpublished paper, Harvard University.

Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay, 1997, *The Econometrics of Financial Markets* (Princeton University Press).

Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–227.

Carroll, Christopher D., Jiri Slacalek, and Martin Sommer, 2008, International Evidence on Sticky Consumption Growth, *SSRN eLibrary.*

Christiano, Larry, 1984, A critique of conventional treatments of the model timing interval in applied econometrics, Manuscript, Federal Reserve Bank of Minneapolis.

——— , and David Marshall, 1987, A note on the estimation of continuous time models from temporally averaged data, Manuscript, Federal Reserve Bank of Minneapolis.

Christiano, Lawrence J., Martin Eichenbaum, and David Marshall, 1990, The permanent income hypothesis revisited, Discussion paper.

Colacito, Riccardo, and Mariano Croce, 2005, Risks for the long run and the real exchange rate, Discussion paper.

Constantinides, George M., and Anisha Ghosh, 2008, Asset Pricing Tests with Long Run Risks in Consumption Growth, *SSRN eLibrary*.

Croce, Mariano M., Martin Lettau, and Sydney C. Ludvigson, 2007, Investor information, long-run risk, and the duration of risky cash-flows, NBER Working Papers 12912 National Bureau of Economic Research, Inc.

Dreschler, Itamar, and Amir Yaron, 2008, Whats vol got to do with it, Unpublished manuscript.

Epstein, Lawrence, and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

——— , 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Geweke, J., 1978, Temporal aggregation in the multiple regression model, *Econometrica* 46, 643–661.

Gultekin, N. Bulent, 1983, Stock market returns and inflation: Evidence from other countries, *Journal of Finance* 38, 49–65.

Hansen, Lars, John Heaton, and Nan Li, 2008, Consumption strikes back? measuring long run risk, *Journal of Political Economy* 116, 260–302.

Hansen, Lars Peter, and Thomas J. Sargent, 2007, Fragile beliefs and the price of model uncertainty., Unpublished manuscript.

Heaton, John, 1993, The interaction between time-nonseparable preferences and time aggregation, *Econometrica* 61, 353–385.

Kan, Raymond, and Xiaolu Wang, 2008, On the Distribution of the Sample Autocorrelation Coefficients, *SSRN eLibrary*.

Kiku, Dana, 2006, Is the value premium a puzzle?, USC FBE Seminar Series Paper.

Malloy, Christopher, Tobias Moskowitz, and Annette Vissing-Jørgensen, 2009, Long-run stockholder consumption risk and asset returns, Forthcoming, Journal of Finance.

Marakani, Srikant, 2009, A trend-stationary long run risk model, Working paper.

Newey, W. K., and K. D. West, 1994, Automatic lag selection in covariance matrix estimation, *Review of Economic Studies* 61, 631–653.

Rietz, Thomas, 1988, The equity premium : a solution, *Journal of Monetary Economics* 22, 117–131.

Sims, C. A., 1971, Discrete approximations to continuous time distributed lags in econometrics, *Econometrica* 39, 545–563.

Wilcox, David, 1992, The construction of the u.s. consumption data : Some facts and their implications for empirical work, *American Economic Review* 82, 922–941.

Working, Holbrook, 1960, Note on the correlation of first differences of averages in a random chain, *Econometrica* 28, 916–918.

Zeileis, Achim, 2006, Implementing a class of structural change tests: An econometric computing approach, *Computational Statistics & Data Analysis* 50, 2987–3008.

——— , Christian Kleiber, Walter Krämer, and Kurt Hornik, 2003, Testing and dating of structural changes in practice, *Computational Statistics & Data Analysis* 44, 109–123.

Zeileis, Achim, Friedrich Leisch, Kurt Hornik, and Christian Kleiber, 2002, strucchange: An r package for testing for structural change in linear regression models, *Journal of Statistical Software* 7, 1–38.

A The BY-LRCR model in more detail

I provide a brief description of the Bansal-Yaron LRCR model in this appendix. The presentation below necessarily closely follows that in Bansal and Yaron Bansal and Yaron (2004).

In this model, the log per capita consumption and per share dividend growth rates g and g_d and their persistent component x are assumed to follow the processes (Bansal and Yaron (2004))

$$x_{t+1} = \rho x_t + \varphi_x \sigma_t e_{t+1} \quad (45)$$

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \quad (46)$$

$$g_{d,t+1} = \mu_d + \phi x_t + \pi_d \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1} \quad (47)$$

$$\sigma^2_{t+1} = \sigma^2 + \nu(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \quad (48)$$

where the shocks e_{t+1}, η_{t+1}, u_{t+1} and w_{t+1} are taken to be independent standard normals[25] for parsimony. Note that μ can be different from μ_d and the consumption and dividend processes are *not* cointegrated[26].

Consumer preferences are assumed to be of the recursive Epstein-Zin (Epstein and Zin (1989)) form. When agents have these preferences, asset returns satisfy

$$E_t[\delta^\theta G_{t+1}^{-\theta/\psi} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \tag{49}$$

where G is the growth rate of per capita consumption, R_a is the gross return on an asset that provides a dividend of per capita consumption, R_i is the asset return, $0 < \delta < 1$ is the time discount factor, γ is the relative risk aversion, ψ is the inter-temporal elasticity of substitution (IES) and θ is defined to be

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}} \tag{50}$$

Note that R_a is unobservable but the asset return R_i is. To use this form of the consumer preference relation, it is therefore needed to relate R_a to observable quantities. This is done by Bansal and Yaron using the standard approximation of Campbell and Shiller (Campbell and Shiller (1988))

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \tag{51}$$

where lowercase letters refer to the logarithm of the variables represented by the corresponding uppercase letters (i.e. $g_t = \log G_t$, $r_{a,t} = \log R_{a,t}$ etc.), $z = \log(\frac{P}{C})$ is the logarithm of the price to consumption ratio and κ_0 and κ_1 are constants that depend only upon the average value of z which is denote by $\bar{z}$ (explicitly, $\kappa_1 = \frac{\exp \bar{z}}{1+\exp \bar{z}}$ Campbell and Shiller (1988)). The analogous approximation for the logarithm of the market return r_m is

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \tag{52}$$

where z_m is the logarithm of the price to dividend ratio. Since the consumption and dividend growth processes are given by (46) and (47), the dynamics for r_a and r_m can be derived once the dynamics of z and z_m are determined. This is done for Epstein-Zin preferences in Bansal and Yaron (2004) where the solution is found to be

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \tag{53}$$

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 \tag{54}$$

[25] In Bansal and Yaron (2004), $\pi_d = 0$. Here, I follow the straightforward extension in Kiku (2006).

[26] While this can be considered a reasonable approximation for the data and sample sizes used in the paper's analysis, this does have the theoretical problem that if I go far enough forward or backward in time, the consumption to dividend ratio will become arbitrarily large or small.

with

$$
\begin{aligned}
A_0 &= \frac{\log\delta + \mu(1-1/\psi) + \kappa_0 + \kappa_1 A_2 (1-\nu)\sigma^2 + \frac{\theta}{2}\kappa_1^2 A_2^2 \sigma_w^2}{1-\kappa_1} && (55)\\
A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} && (56)\\
A_2 &= \frac{\frac{\theta}{2}\left[(1-1/\psi)^2 + (A_1\kappa_1\varphi_x)^2\right]}{1-\kappa_1\nu} && (57)\\
A_{0,m} &= \frac{\Gamma_0 + \kappa_{0,m} + \mu_d + A_{2,m}\kappa_{1,m}(1-\nu)\sigma^2 + \frac{1}{2}(A_{2,m}\kappa_{1,m} - \lambda_w)^2\sigma_w^2}{1-\kappa_{1,m}} && (58)\\
A_{1,m} &= \frac{\phi - 1/\psi}{1-\kappa_{1,m}\rho} && (59)\\
A_{2,m} &= \frac{(1-\theta)A_2(1-\kappa_1\nu) + H_m/2}{1-\kappa_{1,m}\nu} && (60)
\end{aligned}
$$

where Γ_0 and λ_w are defined in (64) and (69) below and H_m is a quantity that is related to the market prices of risk by

$$
H_m = (\gamma - \pi_d)^2 + (\kappa_{1,m} A_{1,m}\varphi_x - \lambda_e)^2 \qquad (61)
$$

where λ_e is the market price of risk corresponding to the shock e (68).

(49) implies that

$$
m_{t+1} = \theta \log\delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \qquad (62)
$$

where m stands for the logarithm of the stochastic discount factor (SDF). (46), (45), (53), (62) and $r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}$ together imply that

$$
m_{t+1} = \Gamma_0 + \Gamma_1 x_t + \Gamma_2 \sigma_t^2 - \lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \sigma_t e_{t+1} - \lambda_w \sigma_w w_{t+1} \qquad (63)
$$

where the coefficients Γ_i are given by

$$
\begin{aligned}
\Gamma_0 &= \log\delta - \frac{\mu}{\psi} - \frac{\theta(\theta-1)}{2}\kappa_1^2 A_2^2 \sigma_w^2 && (64)\\
\Gamma_1 &= -\frac{1}{\psi} && (65)\\
\Gamma_2 &= (\theta - 1)A_2(\kappa_1\nu - 1) && (66)
\end{aligned}
$$

and where the market prices of risk corresponding to the different shocks are given by

$$
\begin{aligned}
\lambda_\eta &= \gamma && (67)\\
\lambda_e &= \left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_1\varphi_x}{1-\kappa_1\rho} && (68)\\
\lambda_w &= (1-\theta)A_2\kappa_1 && (69)
\end{aligned}
$$

The return premium for an asset whose dividend growth process is (47) is thus

$$\begin{aligned} E_t[r_{m,t+1} - r_{f,t+1}] =& \beta_{m,\eta}\lambda_\eta\sigma_t^2 + \beta_{m,e}\lambda_e\sigma_t^2 + \\ & \beta_{m,w}\lambda_w\sigma_w^2 - \frac{1}{2}\mathrm{var}_t(r_{m,t+1}) \end{aligned} \tag{70}$$

where $r_{f,t+1}$ is the logarithm of the risk-free rate from t to $t+1$ and the factor loadings β_m for the asset are

$$\beta_{m,\eta} = \pi_{m,d} \tag{71}$$

$$\beta_{m,e} = \kappa_{1,m}A_{1,m}\varphi_x \tag{72}$$

$$\beta_{m,w} = \kappa_{1,m}A_{2,m} \tag{73}$$

The logarithm of the risk-free rate in the Bansal-Yaron LRCR model is readily derived from the SDF (63) and can be expressed as

$$r_{f,t+1} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 \tag{74}$$

where

$$A_{0,f} = -(\Gamma_0 + \frac{1}{2}\lambda_w^2\sigma_w^2) \tag{75}$$

$$A_{1,f} = \frac{1}{\psi} \tag{76}$$

$$A_{2,f} = -(\Gamma_2 + \frac{1}{2}(\lambda_e^2 + \lambda_\eta^2)) \tag{77}$$

The necessity of Epstein-Zin preferences is evident from the factors of $\gamma - 1/\psi$ in (68) and $1-\theta = \frac{1/\psi-\gamma}{1-1/\psi}$ in (69) which are zero for expected utility preferences as they satisfy the relation $1/\psi = \gamma$ (this is an artifact of their inbuilt assumption that consumers treat the distribution of consumption across time in the same way that they do the distribution across states of nature). Value and small cap stocks within this model are characterized by significantly greater exposure to the persistent long run risk shocks e, or, in other words, by a high factor loading β_e (i.e., a high ϕ). The return premium commanded by the asset for this exposure is given by $\beta_e\lambda_e\sigma_t^2$ which includes the factor $\frac{(\phi-1/\psi)(\gamma-1/\psi)}{(1-\kappa_1\rho)(1-\kappa_{1,m}\rho)}$. Since $\kappa_{1,m} \sim 0.997$ for reasonable preference parameters and from data (on a monthly time scale), this premium is large when $\rho \sim 1$.

B Time aggregation

In this section, I present a pedagogical exposition of the impact of time aggregation on the measured autocorrelation and Granger causality properties of consumption.[27] These effects have been extensively studied in the literature and the results below are not new but are presented here in a manner conducive to an understanding of their importance for the results in the paper.

[27]Breeden, Gibbons, and Litzenberger (1989) has an excellent exposition on this subject

B.1 Effects of time aggregation : a simple linear example

Assume that the consumption rate (or any other flow variable) follows

$$c_{t+\frac{1}{n}} = c_t + \frac{\mu}{n} + \sigma\left(W_{t+\frac{1}{n}} - W_t\right) = c_t + \frac{\mu}{n} + \sigma\Delta W_{t+\frac{1}{n}} \tag{78}$$

where W_t is a standard Wiener process. I am interested in the effects of time aggregation caused due to consumption being measured over a much coarser time scale than the underlying consumption rate process. Since I have assumed the time scale of the underlying process to be $\frac{1}{n}$, I set the time scale of measurement as 1 period.[28]

The measured consumption starting from time T is[29]

$$\begin{aligned} C_T &= \frac{1}{n}\sum_{i=nT+1}^{n(T+1)} c_{i/n} = \frac{1}{n}\sum_{i=nT+1}^{n(T+1)}\left(c_0 + \sigma\sum_{j=1}^{i}\Delta W_{j/n} + \frac{i\mu}{n}\right) \\ &= c_0 + \mu\left(T + \frac{n+1}{2n}\right) \\ &\quad + \sigma\left(\sum_{i=1}^{nT+1}\Delta W_{i/n} + \sum_{i=nT+2}^{n(T+1)}\frac{n(T+1)-i+1}{n}\Delta W_{i/n}\right) \end{aligned} \tag{79}$$

where C_T denotes aggregated consumption starting at time T. Hence,

$$C_0 = c_0 + \sigma\sum_{i=1}^{n}\frac{n-i+1}{n}\Delta W_{i/n} + \frac{n+1}{2n}\mu \tag{80}$$

and

$$\begin{aligned} C_T - C_0 = &\mu T + \sigma\left(\sum_{i=2}^{n}\frac{i-1}{n}\Delta W_{i/n} + \sum_{i=n+1}^{nT+1}\Delta W_{i/n} + \right. \\ &\left. \sum_{i=nT+2}^{n(T+1)}\frac{n(T+1)-i+1}{n}\Delta W_{i/n}\right) \end{aligned} \tag{81}$$

which is a weighted average of consumption innovations from time 2 to time $T+n$, with the weights following an approximately trapezoidal pattern (increasing linearly from $\frac{2}{n}$ to $1+\frac{1}{n}$, remaining constant from $1+\frac{1}{n}$ to $T+\frac{1}{n}$ and then decreasing linearly from $T+\frac{1}{n}$ to $T+1$).

[28] I am not using log consumption, as is conventional, to make the intuition clear since the log case works in the same way after making a log-linear approximation.

[29] The $\frac{1}{n}$ comes about because c_t is the consumption *rate* so that in a time period $\frac{1}{n}$, the consumption is $\frac{c_t}{n}$.

B.2 Exponential growth

Since real per capita consumption exhibits an exponential growth in the data and since it is more natural to think in terms of relative increases in consumption, it is more usual to deal with the logarithm of consumption and to calculate its growth by taking the difference of this series. It is also more conventional to model the dynamics of the logarithm of consumption for the same reasons. I show here that if the consumption series is not too noisy, the results of the previous section go through without any significant modification.[30]

Consider the following process for the log consumption

$$c_{t+\frac{1}{n}} = c_t + \frac{\mu}{n} + \sigma \Delta W_{t+\frac{1}{n}} \tag{82}$$

where W is a standard Wiener process as above and $\Delta W_t = W_t - W_{t-\frac{1}{n}}$. The measured log consumption over n periods starting at time T is then

$$\begin{aligned} C_T = \log n \sum_{i=nT+1}^{n(T+1)} e^{c_{i/n}} = c_{T+\frac{1}{n}} - \log n + \frac{\mu}{2} \\ + \log \sum_{i=nT+1}^{n(T+1)} \exp \left(\left(i - nT - 1 - \frac{n}{2} \right) \frac{\mu}{n} + \sigma \sum_{j=nT+2}^{i} \Delta W_{j/n} \right) \end{aligned} \tag{83}$$

If μ and v are small, the last term is approximately

$$\begin{aligned} &\log \sum_{i=nT+1}^{n(T+1)} \left(1 + \left(i - nT - 1 - \frac{n}{2} \right) \frac{\mu}{n} + \sigma \sum_{j=nT+2}^{i} \Delta W_{j/n} \right) \\ &\approx \log \left(n - \frac{\mu}{2} + \sum_{i=nT+2}^{n(T+1)} \sigma \sum_{j=nT+2}^{i} \Delta W_{j/n} \right) \\ &= \log n + \log \left(1 - \frac{\mu}{2n} + \frac{\sigma}{n} \sum_{i=nT+2}^{n(T+1)} (n(T+1) + 1 - i) \Delta W_{i/n} \right) \\ &\approx \log n - \frac{\mu}{2n} + \sigma \sum_{i=nT+2}^{n(T+1)} \frac{n(T+1) + 1 - i}{n} \Delta W_{i/n} \end{aligned} \tag{84}$$

[30] There is a change in the specification of the process over time scale due to the different additive natures of exponential and linear processes but it is a fairly trivial one.

so that

$$\begin{aligned} C_T \approx & c_{T+\frac{1}{n}} + \frac{n-1}{2n}\mu + \sigma \sum_{i=nT+2}^{n(T+1)} \frac{n(T+1)+1-i}{n} \Delta W_{i/n} \\ = & c_{\frac{1}{n}} + \left(T + \frac{n-1}{2n}\right)\mu + \sigma \sum_{i=2}^{nT+1} \Delta W_{i/n} \\ & + \sigma \sum_{i=nT+2}^{n(T+1)} \frac{n(T+1)+1-i}{n} \Delta W_{i/n} \end{aligned} \tag{85}$$

so that the analog of (81) to the simple linear process examined earlier is

$$\begin{aligned} C_T - C_0 \approx & \mu T + \sigma \left(\sum_{i=2}^{n} \frac{i-1}{n} \Delta W_{i/n} + \sum_{i=n+1}^{nT+1} \Delta W_{i/n} \right. \\ & \left. + \sum_{i=nT+2}^{n(T+1)} \frac{n(T+1)+1-i}{n} \Delta W_{i/n} \right) \end{aligned} \tag{86}$$

which leads to the *same* continuous time approximation as for the simple linear case. Hence, all the results from the simple linear case are applicable to the exponential growth case provided μ and $\sigma \sum_{i=nT+1}^{n(T+1)} \Delta W_{i/n}$ are small. This is the case when considering annual or quarterly consumption since $\mu \approx 0.02$/year and $\sigma \approx 0.018$ on an annual time scale.

B.3 Continuous time

B.3.1 Linear process in continuous time

The above results are easily extended to continuous time by making v/σ a standard Brownian motion and by letting μ be the growth rate over one period which I keep to be the unit of measurement for consumption. With this reinterpretation, the continuous consumption process is

$$dc_t = \mu dt + \sigma dW(t) \tag{87}$$

or

$$c_t = c_0 + \mu t + \sigma W(t) \tag{88}$$

Relation (79) now becomes

$$\begin{aligned} C_T = & \int_T^{T+1} c_t dt = c_0 + \frac{T+1}{2}\mu + \sigma \int_T^{T+1} dt \int_0^t dW(t') \\ = & c_0 + \frac{T+1}{2}\mu + \sigma \left(\int_T^{T+1} dt \int_0^T dW(t') + \int_{t'}^{T+1} dt \int_T^{T+1} dW(t') \right) \\ = & c_0 + \frac{T+1}{2}\mu + \sigma \left(\int_0^T dW(t) + \int_T^{T+1} (T+1-t) dW(t) \right) \end{aligned} \tag{89}$$

where c_0 is the analogous consumption rate at time 0 and (81) (with T now understood to be T time units) becomes

$$C_T - C_0 = \mu T + \sigma \left(\int_0^1 t dW(t) + \int_1^T dW(t) + \int_T^{T+1} (T+1-t) dW(t) \right) \quad (90)$$

For future analysis, I note the useful specialization of the above formula

$$C_{t+1} - C_t = \mu + \sigma \left(\int_{t-1}^t (t' - t + 1) dW(t') + \int_t^{t+1} (t+1-t') dW(t') \right) \quad (91)$$

B.3.2 Continuous time version of the exponential growth model

From the discussion above, it is evident that to a very good approximation, the aggregated consumption growth in the discrete version of the exponential growth model (82) reduces to (86) which is *exactly* the same as in the linear growth model (81). Hence, the continuous time limit of the exponential growth model is the *same* as that for the linear growth model. Hence, everything derived above for the continuous time version of the linear growth model applies to the exponential growth model with no change whatsoever. In particular (90) holds, and therefore all the results derived from it also apply to the exponential growth model.

B.3.3 Autocorrelations of aggregated processes

Expression (90) allows easy calculation of correlations between measured consumption growths. For example, the correlation between $C_8 - C_4$ and $C_4 - C_0$ (analog of consumption growth between the same quarter in successive years) is

$$\text{corr}(C_8 - C_4, C_4 - C_0) = \frac{\sigma^2 (\int_4^5 (t-4)(5-t) dt)}{\sigma^2 (\int_0^1 t^2 dt + \int_1^4 dt + \int_4^5 (5-t)^2 dt)} = \frac{\frac{1}{6}}{\frac{11}{3}} = \frac{1}{22} \quad (92)$$

which, while small, is not zero.[31] Similarly, the correlation between $C_1 - C_0$ and $C_2 - C_1$ is $1/4$, which is a well known result (Working (1960)).

B.3.4 Predictability

From (90), it is seen that the information set $\mathcal{F}_t$ for any $0 < t < T + 1$, can be used to predict part of $C_T - C_0$. In other words,

$$\begin{aligned} E[C_T - C_0 | \mathcal{F}_t] = & \int_0^{\min(t,1)} t' dW(t') + \int_1^{\min(t,T)} dW(t') \\ & + \int_T^{\min(t,T+1)} (T+1-t') dW(t') \end{aligned} \quad (93)$$

[31] The above calculation makes use of the obvious fact that the variances of $C_4 - C_0$ and $C_8 - C_4$ are equal.

Hence, use of the information set $\mathcal{F}_t$ with $t > 0$ will show predictability for $C_T - C_0$ even though the underlying consumption growth is unpredictable. To show true predictability of the underlying consumption growth, an information set $\mathcal{F}_t$ with $t < 0$, or more generally, before the start of the period from which the initial consumption is measured, should be used.

To see how much effect this can have on the predictability of measured one period consumption growth, I calculate from (91) that

$$\frac{\mathrm{Var}(E_t[C_{t+1} - C_t])}{\mathrm{Var}(C_{t+1} - C_t)} = \frac{\mathrm{Var}\left(\int_{t-1}^{t}(t'-t+1)dW(t')\right)}{\mathrm{Var}\left(\int_{t-1}^{t}(t'-t+1)dW(t') + \int_{t}^{t+1}(t+1-t')dW(t')\right)} = \frac{1}{2} \tag{94}$$

In other words, about 50% of the measured one period consumption growth would appear to be predictable using information available at time t! This high level of spurious predictability implies that one should be very careful to only use information at time $t-1$ when making claims about the predictability of consumption growth from time t to $t+1$.

In some previous studies, it has been claimed that taking into account the spurious autocorrelation of residuals that occurs due to time aggregation is also sufficient to remove the related spurious predictability. I now show that this is not the case.

The spurious autocorrelation is generally taken into account by specifying that the residuals follow a MA(1) type process

$$C_{t+1} - C_t = \mu + \beta X_t + \sigma(\eta_{t+1} + \omega\eta_t) \tag{95}$$

where X are the predictors, $E_{t-1}[\eta_{t+1}] = E_{t-1}[\eta_t] = 0$, $\eta_t \in \mathcal{F}_t$, $\mathrm{Cov}(\eta_{t+1}, \eta_t) = 0$ and where $\omega = 2 - \sqrt{3}$ would set the autocorrelation of the residuals to $\frac{1}{4}$ if η is homoskedastic, the expected value in the case of no predictability and continuous time aggregation. In practice, ω is fitted as the precise time scale of the aggregation is uncertain.

Under the null of no predictability and homoskedasticity, this gives

$$C_{t+1} - C_t = \mu + \sigma(\eta_{t+1} + \omega\eta_t) \tag{96}$$

To get this to agree with (94), I need

$$\frac{\mathrm{Var}(E_t[C_{t+1} - C_t])}{\mathrm{Var}(C_{t+1} - C_t)} = \frac{E[(E_t[\eta_{t+1}])^2 + \omega^2\eta_t^2]}{E[\eta_{t+1}^2 + \omega^2\eta_t^2]} = \frac{1}{2} \tag{97}$$

(where I have used $\mathrm{Cov}(\eta_{t+1}, \eta_t) = 0$) to be true.

Using this result, I now show that $E_t[\eta_{t+1}] \neq 0$ under the null of no predictability and homoskedasticity. The process η is homoskedastic under our assumptions since $\mathrm{Var}(C_{t+k+1} - C_{t+k}) = \sigma^2(\mathrm{Var}(\eta_{t+k+1}) + \omega^2\mathrm{Var}(\eta_{t+k}))$ is independent of k. If $E_t[\eta_{t+1}] = 0$, (94) implies that $\omega = 1$ which leads to a counter-factually high first order autocorrelation of $\frac{1}{2}$. Hence, we have by contradiction that $E_t[\eta_{t+1}] \neq 0$.

I now analyze the structure of $C_{t+1} - C_t$ to find a good representation analogous to (95). I first disassociate the parts of $C_{t+1} - C_t$ which are predictable and unpredictable at time t by using (91) and writing

$$C_{t+1} - C_t = \mu + \sigma(\eta^a_{t+1} + \eta^b_t) \tag{98}$$

where

$$\eta^a_t = \int_{t-1}^{t} (t - t')dW(t') \tag{99}$$

$$\eta^b_t = \int_{t-1}^{t} (t' - t + 1)dW(t') \tag{100}$$

Clearly $E_t[\eta^a_{t+1}] = E_t[\eta^b_{t+1}] = 0$, $\eta^b_t \in \mathcal{F}_t$, η^a and η^b are each i.i.d with the same distribution, and η^a_t and η^b_{t+k} are uncorrelated for all $k \neq 0$. I note that $\eta^a \neq \eta^b$ and therefore, if I want to write (98) in a form similar to that of (96), I can make use of the fact that $\text{Corr}(\eta^a_t, \eta^b_t) = \frac{1}{4}$ to write $\eta^b_t = \frac{1}{2}\eta^a_t + \frac{\sqrt{3}}{2}\epsilon_t$ where η^a and ϵ are i.i.d with the same distribution $(N(0, \frac{1}{3}))$ and uncorrelated at all lags. Hence, I can finally write

$$C_{t+1} - C_t = \mu + \sigma(\eta_{t+1} + \omega\eta_t + \alpha\epsilon_t) \tag{101}$$

where I drop the superfluous superscript, and where $E_t[\eta_{t+1}] = 0$, $E_{t-1}[\epsilon_t] = 0$ and $\eta_t, \epsilon_t \in \mathcal{F}_t$. For continuous time aggregation with homoskedasticity, $\omega = \frac{1}{2}$, $\alpha = \frac{\sqrt{3}}{2}$ and η, ϵ are i.i.d. normal processes with variance $\frac{1}{3}$. Hence, any predictability equation must be written as

$$C_{t+1} - C_t = \mu + \beta X_t + \sigma(\eta_{t+1} + \omega\eta_t + \alpha\epsilon_t) \tag{102}$$

This implies that any estimation procedure for β would have to take into account the extra term $\epsilon_t \in \mathcal{F}_t$. This is easily taken care of if $X_t \in \mathcal{F}_{t-1}$ as the error term is then uncorrelated with the predictor. However, if this is not the case, I know of no easy way to account for the endogenous and unobserved term ϵ_t above.

I now show that the result above generalizes quite naturally to the heteroskedastic case. If the consumption process is

$$dc_t = \mu dt + \sigma_t dW(t) \tag{103}$$

then the aggregated consumption starting at time T is given by

$$\begin{aligned} C_T = & \int_T^{T+1} c_t dt = c_0 + \frac{T+1}{2}\mu + \int_T^{T+1} dt \int_0^t \sigma_{t'} dW(t') \\ = & c_0 + \frac{T+1}{2}\mu + \int_T^{T+1} dt \int_0^T \sigma_{t'} dW(t') + \int_{t'}^{T+1} dt \int_T^{T+1} \sigma_{t'} dW(t') \\ = & c_0 + \frac{T+1}{2}\mu + \int_0^T \sigma_t dW(t) + \int_T^{T+1} (T + 1 - t)\sigma_t dW(t) \end{aligned} \tag{104}$$

and

$$\begin{aligned} C_{t+1} - C_t &= \mu + \int_{t-1}^{t} (t' - t + 1)\sigma_{t'} dW(t') + \int_{t}^{t+1} (t + 1 - t')\sigma_{t'} dW(t') \\ &= \mu + \sigma_{b,t}\eta_t^b + \sigma_{a,t+1}\eta_{t+1}^a \end{aligned} \tag{105}$$

where $\sigma_{a,t}^2 = \int_{t-1}^{t} (t - t')^2 \sigma_{t'}^2 dt$, $\sigma_{b,t}^2 = \int_{t-1}^{t} (t' - t + 1)\sigma_{t'}^2 dt'$ and η^a, η^b are i.i.d. standard normal processes.

The correlation between $C_{t+2} - C_{t+1}$ and $C_{t+1} - C_t$ is now given by

$$\mathrm{Corr}(C_{t+1} - C_t, C_t - C_{t-1}) = \tilde{\omega}_t = \frac{\int_{t-1}^{t} (t - t')(t' - t + 1)\sigma_{t'}^2 dt'}{\tilde{\sigma}_t \tilde{\sigma}_{t+1}} \tag{106}$$

where

$$\tilde{\sigma}_{t+1}^2 = \int_{t-1}^{t} (t' - t + 1)^2 \sigma_{t'}^2 dt' + \int_{t}^{t+1} (t + 1 - t')^2 \sigma_{t'}^2 dt' = \sigma_{b,t}^2 + \sigma_{a,t+1}^2 \tag{107}$$

is the volatility of $C_{t+1} - C_t$. In the homoskedastic case, $\tilde{\sigma} = \sqrt{\frac{2}{3}}\sigma$. I can now write the analogue of (101) for the stochastic volatility case as

$$C_{t+1} - C_t = \mu + \sigma_{a,t+1}\eta_{t+1}^a + \sigma_{a,t}(\omega_t \eta_t^a + \alpha_t \epsilon_t) \tag{108}$$

where $\omega_t \sigma_{a,t}^2 = \tilde{\omega}_t \tilde{\sigma}_t \tilde{\sigma}_{t+1}$, $\alpha_t = \frac{\sigma_{b,t}}{\sigma_{a,t}} \sqrt{1 - \omega_t^2}$ and η^a, ϵ are i.i.d. standard normal processes. I note that the extra term ϵ is still required and that ω and α must now be time varying. In practice, ω_t and α_t depend on the time scale of aggregation and would have to be independently estimated. The predictability equation now becomes

$$C_{t+1} - C_t = \mu + \beta X_t + \sigma_{a,t+1}\eta_{t+1}^a + \sigma_{a,t}(\omega_t \eta_t^a + \alpha_t \epsilon_t) \tag{109}$$

and any estimation procedure must take into account the process ϵ_t if $X_t \in \mathcal{F}_t$ and $X_t \notin \mathcal{F}_{t-1}$. Since ϵ_t is not observed, I know of no way to do so.

C Time aggregation and the Bansal-Yaron model

The Bansal-Yaron model postulates that

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \tag{110}$$

$$x_{t+1} = \rho x_t + \varphi_x \sigma_t e_{t+1} \tag{111}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \tag{112}$$

$$g_{d\,t+1} = \mu_d + \phi x_t + \pi_d \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1} \tag{113}$$

where Δc and g_d are consumption and dividend growth and η, e, u and w are i.i.d. standard normal processes. The process is specified on the monthly level but the data used is on an annual time scale. Hence, time aggregation plays a big part in the analysis and estimation of this model.

Bansal, Yaron, and Kiku (2007b) derive the time aggregated equation (note that the time unit here is one *month*)

$$\begin{aligned} \Delta c^a_{12(t+1)} &= 12\mu + \phi^a_c x_{12(t+1)-23} + \eta^a_{12(t+1)} && (114) \\ x_{12(t+1)} &= \rho^{12} x_{12t} + \varphi_x e^a_{12(t+1)} && (115) \end{aligned}$$

(the superscript ^{a} denotes aggregated quantities) where Δc^a is the measured aggregated annual consumption growth and where

$$\phi^a_c = \sum_{j=0}^{22} \tau_j \rho^j \quad (116)$$

$$\eta^a_{12(t+1)} = \sum_{j=0}^{22} \tau_j \Bigg(\sigma_{12(t+1)-j-1} \eta_{12(t+1)-j} + \varphi_x \sum_{k=1}^{22-j} \rho^{k-1} \sigma_{12(t+1)-j-1-k} e_{12(t+1)-j-k} \Bigg) \quad (117)$$

$$e^a_{12(t+1)} = \sum_{j=0}^{11} \rho^j \sigma_{12(t+1)-j-1} e_{12(t+1)-j} \quad (118)$$

with τ_j being given by

$$\tau_j = \begin{cases} \frac{j+1}{12} & j < 12 \\ \frac{23-j}{12} & j \geq 12 \end{cases} \quad (119)$$

To be consistent with my notation and time unit, I write the equivalent set of equations with the new time unit

$$\begin{aligned} \Delta c^a_{t+1} &= \mu + \rho \phi^a_c x_{t-1} + \eta^a_{t+1} && (120) \\ x_{t+1} &= \rho^{12} x_t + \varphi_x e^a_{t+1} && (121) \end{aligned}$$

where

$$\phi^a_c = \sum_{j=0}^{22} \tau_j \rho^j \quad (122)$$

$$\eta^a_{t+1} = \sum_{j=0}^{22} \tau_j \Bigg(\sigma_{t+1-\frac{j+1}{12}} \eta_{t+1-\frac{j}{12}} + \varphi_x \sum_{k=1}^{23-j} \rho^{k-1} \sigma_{t+1-\frac{j+k+1}{12}} e_{t+1-\frac{j+k}{12}} \Bigg) \quad (123)$$

$$e^a_{t+1} = \sum_{j=0}^{11} \rho^j \sigma_{t+1-\frac{j+1}{12}} e_{t+1-\frac{j}{12}} \quad (124)$$

The spuriously predictable part of aggregated consumption growth is now

$$E_t[\eta^a_{t+1}] = \sum_{j=12}^{22} \tau_j \sigma_{t+1-\frac{j+1}{12}} \eta_{t+1-\frac{j}{12}} + \varphi_x \sum_{j=0}^{22} \tau_j \sum_{k=\max(1,12-j)}^{23-j} \rho^{k-1} \sigma_{t+1-\frac{j+k+1}{12}} e_{t+1-\frac{j+k}{12}} \tag{125}$$

where $E_{t-1}[\eta^a_{t+1}] = 0$ and $E[\eta^a_{t+1}|x_{t-1}] = 0$.

As noted in Bansal, Yaron, and Kiku (2007b), (23) can also be expressed as

$$\Delta c^a_{t+1} = \mu + \rho^{-11} \phi^a_c x_t + \tilde{\eta}^a_{t+1} \tag{126}$$

$$x_{t+1} = \rho^{12} x_t + \varphi_x e^a_{t+1} \tag{127}$$

where

$$\tilde{\eta}^a_{t+1} = \sum_{j=0}^{22} \tau_j \sigma_{t+1-\frac{j+1}{12}} \eta_{t+1-\frac{j}{12}} + \varphi_x \left(\sum_{i=1}^{11} \sum_{j=0}^{i-1} - \sum_{i=12}^{23} \sum_{j=i}^{22} \right) \tau_j \rho^{i-j-1} \sigma_{t+1-\frac{i+1}{12}} e_{t+1-\frac{i}{12}} \tag{128}$$

$$e^a_{t+1} = \sum_{j=0}^{11} \rho^j \sigma_{t+1-\frac{j+1}{12}} e_{t+1-\frac{j}{12}} \tag{129}$$

The spuriously predictable portion is now given by

$$E_t[\tilde{\eta}^a_{t+1}] = \sum_{j=12}^{22} \tau_j \sigma_{t+1-\frac{j+1}{12}} \eta_{t+1-\frac{j}{12}} - \varphi_x \sum_{i=12}^{23} \sum_{j=i}^{22} \tau_j \rho^{i-j-1} \sigma_{t+1-\frac{i+1}{12}} e_{t+1-\frac{i}{12}} \tag{130}$$

where $E_{t-1}[\tilde{\eta}^a_{t+1}] = 0$ and $E[\tilde{\eta}^a_{t+1}|x_t] \neq 0$ since the coefficients on the innovations $e_{t-\frac{i}{12}}$ for $0 \leq i \leq 11$ are non-zero.

Defining $\text{Cov}(\tilde{\eta}^a_t, \tilde{\eta}^a_{t+1}) = \tilde{\omega}_t \tilde{\sigma}_t \tilde{\sigma}_{t+1}$ as before (where $\tilde{\sigma}^2_t = \text{Var}(\tilde{\eta}^a_t)$), we obtain

$$\tilde{\omega}_t \tilde{\sigma}_t \tilde{\sigma}_{t+1} = \sum_{j=0}^{10} \tau_j \tau_{j+12} \sigma^2_{t-\frac{j+1}{12}} - \varphi_x^2 \left(\sum_{i=1}^{11} \left(\sum_{j=0}^{i-1} \tau_j \rho^{i-j-1} \right) \left(\sum_{j=i+12}^{22} \tau_j \rho^{i+11-j} \right) \sigma^2_{t-\frac{i+1}{12}} \right) \tag{131}$$

and

$$\tilde{\sigma}_t^2 = \sum_{j=0}^{11} \tau_j^2 \sigma^2_{t-\frac{j+1}{12}} + \varphi_x^2 \left(\sum_{i=1}^{11} \left(\sum_{j=0}^{i-1} \tau_j \rho^{i-j-1} \right)^2 \sigma^2_{t-\frac{i+1}{12}} \right) + \sum_{j=12}^{22} \tau_j^2 \sigma^2_{t-\frac{j+1}{12}} + \varphi_x^2 \left(\sum_{i=12}^{23} \left(\sum_{j=i}^{22} \tau_j \rho^{i-j-1} \right)^2 \sigma^2_{t-\frac{i+1}{12}} \right) = \sigma^2_{a,t} + \sigma^2_{b,t-1} \tag{132}$$

Using these expressions which are exactly analogous to the continuous time stochastic volatility case considered in the previous section, I can write

$$\Delta c^a_{t+1} = \mu + \rho^{-11} \phi^a_c x_t + \sigma_{a,t+1} \tilde{\eta}^a_{t+1} + \sigma_{a,t}(\omega_t \tilde{\eta}^a_t + \alpha_t \epsilon^a_t) \tag{133}$$

where $\omega_t \sigma^2_{a,t} = \tilde{\omega}_t \tilde{\sigma}_t \tilde{\sigma}_{t+1}$, $\alpha_t = \frac{\sigma_{b,t}}{\sigma_{a,t}} \sqrt{1-\omega_t^2}$ and $\tilde{\eta}^a, \epsilon^a$ are i.i.d. standard normal processes. This is very similar to the expression in the appendix on time aggregation in Bansal, Yaron, and Kiku (2007b) but that expression excludes the time dependence of ω as well as the spuriously predictable term ϵ_t.

The derivation of the above expressions can be straightforwardly generalized to n year consumption growth with the following results

$$c^a_{t+n} - c^a_t = \mu + \rho \phi^n_c x_{t-1} + \eta^n_{t+n} \tag{134}$$

$$c^a_{t+n} - c^a_t = \mu + \rho \phi^n_c (\alpha_0 + \alpha_1 r_{f,t-\frac{11}{12}} + \alpha_2 z_{m,t}) + \eta^n_{t+n} \tag{135}$$

$$x_{t+n} = \rho^{12n} x_t + \varphi_x e^n_{t+1} \tag{136}$$

where

$$\phi^n_c = \sum_{j=0}^{12(n+1)-2} \tau^n_j \rho^j \tag{137}$$

$$\eta^n_{t+n} = \sum_{j=0}^{12(n+1)-2} \tau^n_j \left(\sigma_{t+n-\frac{j+1}{12}} \eta_{t+n-\frac{j}{12}} + \varphi_x \sum_{k=1}^{12(n+1)-j-1} \rho^{k-1} \sigma_{t+n-\frac{j+k+1}{12}} e_{t+n-\frac{j+k}{12}} \right) \tag{138}$$

$$e^n_{t+n} = \sum_{j=0}^{n-1} \rho^j \sigma_{t+n-\frac{j+1}{12}} e_{t+n-\frac{j}{12}} \tag{139}$$

with τ^n_j being given by

$$\tau^n_j = \begin{cases} \frac{j+1}{12} & j < 12 \\ 1 & 12 \le j \le 12n-1 \\ \frac{23-j}{12} & j \ge 12n \end{cases} \tag{140}$$

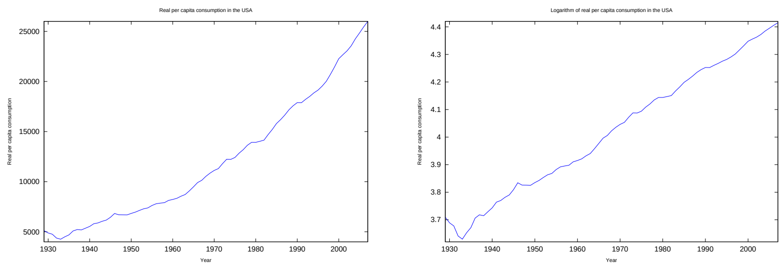

Figure 1: Real per capita consumption in the USA and its logarithm.

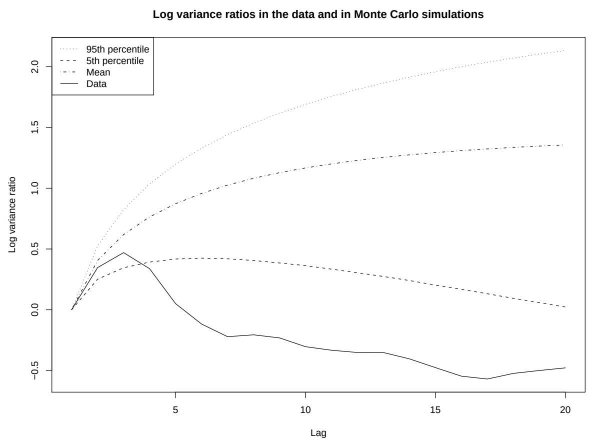

Figure 2: Log variance ratios of consumption growth from 1930-2008 together with the mean, 5th and 95th percentiles implied for them by the calibration in Bansal, Yaron, and Kiku (2007b).

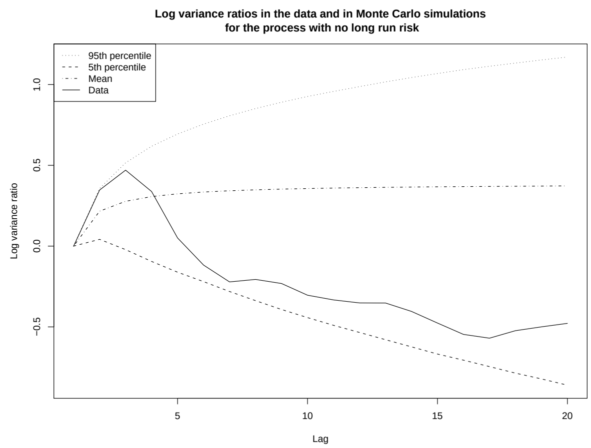

Figure 3: Log variance ratios of consumption growth from 1930-2008 together with the mean, 5th and 95th percentiles implied for them by the the BY-LRCR consumption process with $\varphi_x = 0$, or in other words, with no long run risk.

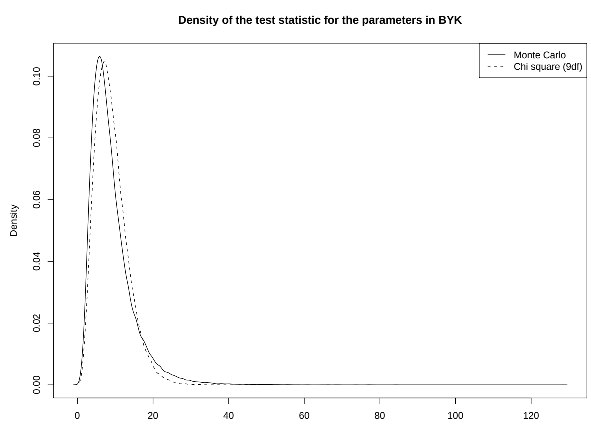

Figure 4: Asymptotic and estimated probability distribution of the test statistic t^2 under the null of the the BY-LRCR model with the parameter values calibrated in Bansal, Yaron, and Kiku (2007b).

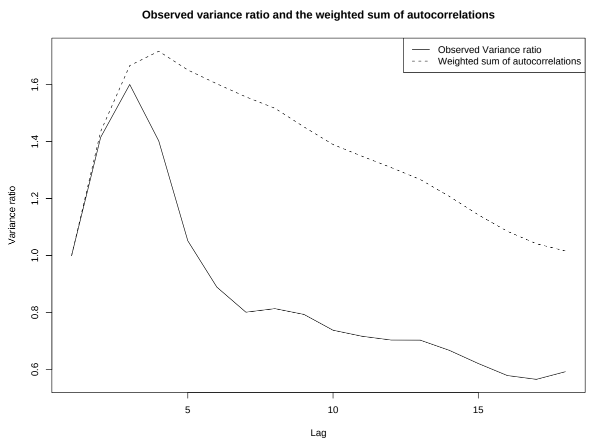

Figure 5: The observed variance ratios and the values obtained using (39).

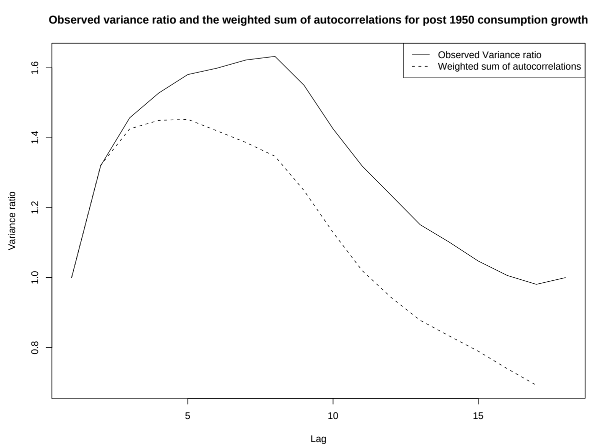

Figure 6: The observed variance ratios of post 1950 consumption growth and the values obtained using (39).

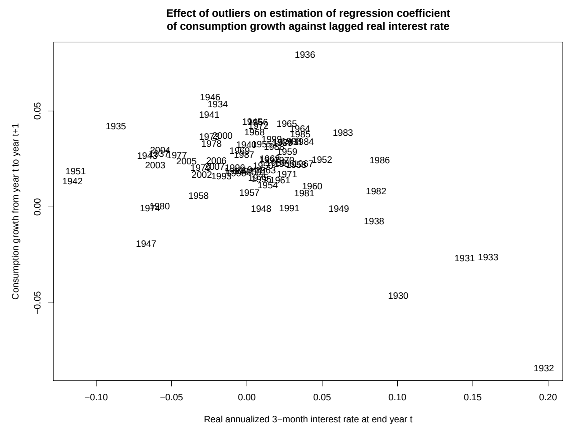

Figure 7: The effect of outliers on the regressions can be clearly seen in this graph of consumption growth against singly lagged real risk free rate with the conversion to real being done using realized annual inflation.

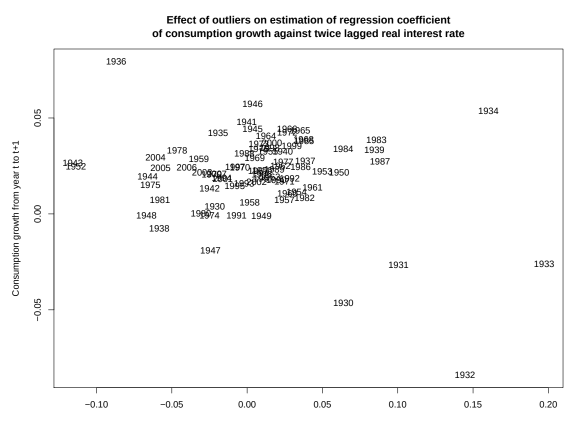

Figure 8: The effect of outliers on the regressions can be clearly seen in this graph of consumption growth against twice lagged real risk free rate with the conversion to real being done using realized annual inflation.

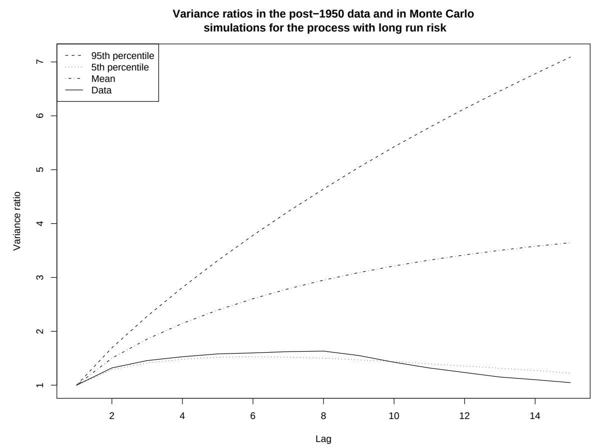

Figure 9: Variance ratios of post-1950 consumption growth together with the mean, 5th and 95th percentiles implied by the BY-LRCR process with the parameter estimates in Bansal, Yaron, and Kiku (2007b).

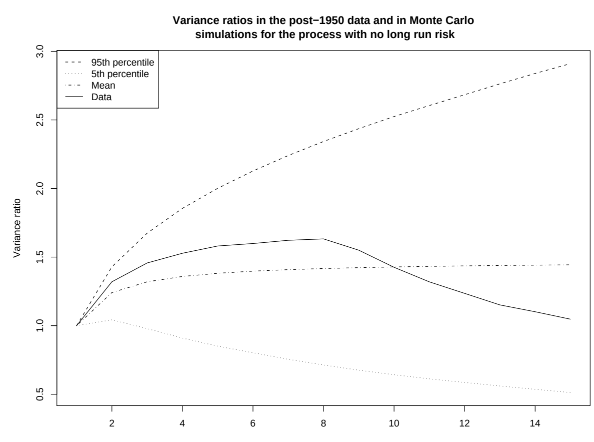

Figure 10: Variance ratios of post-1950 consumption growth together with the mean, 5th and 95th percentiles implied for them by the BY-LRCR consumption process with $\varphi_x = 0$, or in other words, with no long run risk.

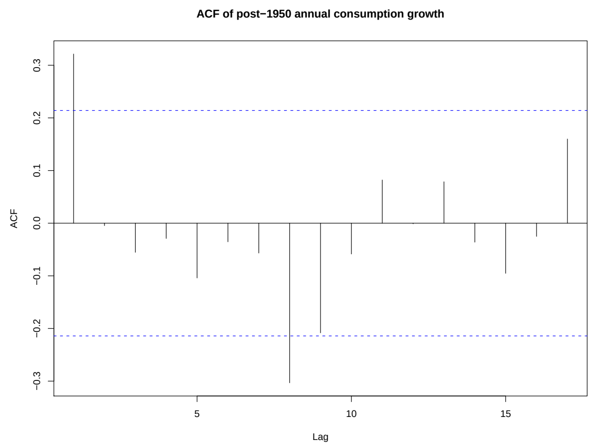

Figure 11: The autocorrelations of post-1950 annual consumption growth.

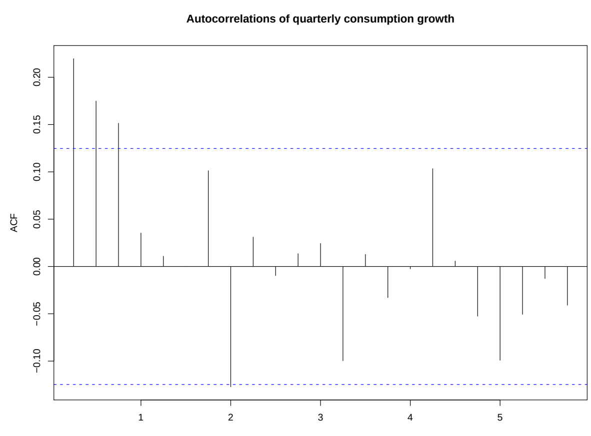

Figure 12: Autocorrelation structure of quarterly consumption growth. The blue lines represent 95% confidence intervals based on the null of a random walk without taking time aggregation into account.

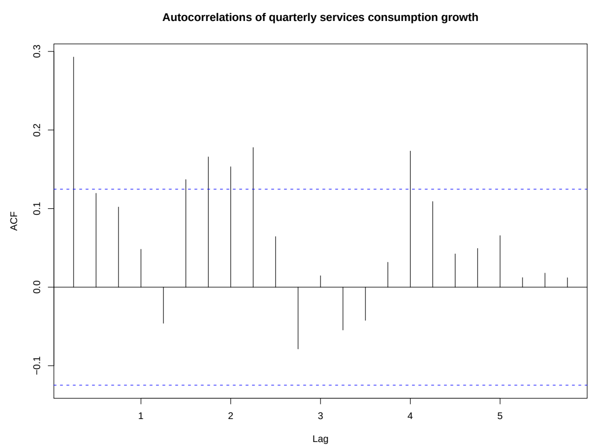

Figure 13: Autocorrelation structure of quarterly services consumption growth. The blue lines represent 95% confidence intervals based on the null of a random walk without taking time aggregation into account.

μ	ρ	φ_x	σ	ν	σ_w	ϕ	π_d	φ_d
0.0015	0.982	0.042	5.4×10^{-3}	0.98	6.8×10^{-6}	2.3	3.8	5.4

Table 1: Parameters estimated in Bansal, Yaron, and Kiku (2007b). The dividend dynamics estimates are for the market portfolio.

Coefficient	Realized annual inflation	Realized quarterly inflation	Lagged annual inflation	Lagged quarterly inflation
$\alpha_0 + \mu$	-0.00080	0.00927	-0.0039	-0.0032
	(0.0175)	(0.0124)	(0.0183)	(0.0191)
α_1	-0.0953	-0.0898	-0.0981	-0.0735
	(0.1362)	(0.0719)	(0.1031)	(0.0941)
α_2	0.0066	0.0036	0.0075	0.0076
	(0.0051)	(0.0036)	(0.0053)	(0.0058)
Adjusted r^2	0.021	0.033	0.018	0.016

Table 2: Predictability of consumption growth by the twice lagged real risk-free rate and log price-dividend ratio using data from 1930 to 2008. The nominal risk-free rate is converted to real using the specified inflation. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994).

Coefficient	Realized annual inflation	Realized quarterly inflation	Lagged annual inflation	Lagged quarterly inflation	Null
$\alpha_0 + \mu$	0.0336	0.0347	0.0334	0.0283	
	(0.0302)	(0.027)	(0.0304)	(0.0276)	
α_1	0.0267	-0.0570	-0.165	-0.0662	
	(0.0835)	(0.0493)	(0.0709)	(0.0518)	
α_2	-0.0041	-0.0042	-0.0037	-0.0020	
	(0.0090)	(0.0080)	(0.0091)	(0.0083)	
ω	0.5279	0.4749	0.620	0.4996	
	(0.1805)	(0.1338)	(0.169)	(0.1378)	
AIC	-386.35	-387.57	-391.21	-387.87	-390.1
Adj. r^2	-0.022	-0.007	0.041	-0.003	0

Table 3: Predictability of consumption growth by the twice lagged real risk-free rate and log price-dividend ratio using the estimation method (29) and data from 1930 to 2008. This estimation method takes the expected MA(1) correlation structure of the residuals into account. The nominal risk-free rate is converted to real using the specified inflation. Standard errors are reported in parentheses.

Coefficient	Realized annual inflation	Realized quarterly inflation	Lagged annual inflation	Lagged quarterly inflation	Null
$\alpha_0 + \mu$	-0.0546	-0.0474	-0.0630	-0.0619	
	(0.0196)	(0.024)	(0.0237)	(0.0229)	
α_1	-0.2372	-0.0746	-0.0111	-0.0476	
	(0.0534)	(0.0547)	(0.0810)	(0.0523)	
α_2	0.0228	0.0204	0.0249	0.0248	
	(0.0058)	(0.0070)	(0.0070)	(0.0068)	
ω	0.4630	0.4162	0.5247	0.4920	
	(0.1228)	(0.1446)	(0.1575)	(0.1314)	
AIC	-416.85	-401.04	-399.28	-400.08	-390.1
Adj. r^2	0.305	0.150	0.132	0.140	0

Table 4: Predictability of consumption growth by the singly lagged real risk-free rate and log price-dividend ratio using the *inconsistent* estimation method (95) and data from 1930 to 2008. The nominal risk-free rate is converted to real using the specified inflation.

Coefficient	Estimate	Std. error
$\alpha_0 + \mu$	0.0015	0.0173
α_1	-0.0004	0.0033
α_2	0.0061	0.0051

Table 5: Predictability of consumption growth by the twice lagged log value price-dividend ratio and log market price-dividend ratio using data from 1930 to 2008. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994). The adjusted r^2 is found to be -0.011.

Coefficient	Estimate	Std. error
$\alpha_0 + \mu$	0.01956	0.00267
α_1	0.00157	0.00231
α_2	0.00163	0.00319
α_3	-0.0038	0.0111

Table 6: Predictability of annual consumption growth by the first three principal components of the log price-dividend ratios of the six Fama-French portfolios sorted on the basis of size and book to market ratio. The values are lagged twice to account for time aggregation and the regression is from 1930 to 2008. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994). The adjusted r^2 is found to be -0.022 (the r^2 is 0.017).

Coefficient	Real risk-free rate and price-dividend ratio	Market and value price-dividend ratios
$\alpha_0 + \mu$	0.0194 (0.0147)	0.0228 (0.0107)
α_1	0.0159 (0.0970)	-0.0088 (0.0055)
α_2	0.0010 (0.0041)	0.0084 (0.0058)
Adjusted r^2	-0.033	0.004

Table 7: Predictability of consumption growth by the twice lagged real risk-free rate and log price-dividend ratio using data from 1950 to 2008. The nominal risk-free rate is converted to real using the annualized median expected six-month inflation from the Livingston survey. The standard errors are Newey-West corrected with the number of lags required estimated using the procedure in Newey and West (1994). For the results using two price-dividend ratios, α_1 corresponds to the coefficient for the log price-dividend ratio of the value portfolio.

Coefficient	Real risk-free rate and price-dividend ratio	Market and value price-dividend ratios	Null
$\alpha_0 + \mu$	0.0300 (0.0153)	0.0348 (0.0152)	
α_1	0.0027 (0.0922)	-0.0143 (0.0067)	
α_2	-0.0020 (0.0044)	0.0100 (0.0071)	
ω	0.3701 (0.1186)	0.4334 (0.1149)	0.3542 (0.1148)
AIC	-362.30	-366.81	-366.08
Adjusted r^2	-0.032	0.045	0

Table 8: Predictability of consumption growth by the twice lagged real risk-free rate and log price-dividend ratio using data from 1950 to 2008 where the regressions have been carried out by taking into account the expected MA(1) nature of the residual autocorrelation. The nominal risk-free rate is converted to real using the annualized median expected six-month inflation from the Livingston survey. For the results using two price-dividend ratios, α_1 corresponds to the coefficient for the log price-dividend ratio of the value portfolio.